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ABSTRACT

Suggestions for using four-function calculators, programmable calculators, and microcomputers are considered in this collection of 36 articles. The first section contains articles considering general implications for mathematics curricula implied by the freedom calculators offer students from routine computation, enabling them to focus on results and relationships, and is balanced by Section Two, exploring inappropriate ways calculators can be used. Freedom from thinking about routine calculations provides freedom for thinking about problem solving is the theme of Section Three. Articles in Section Four include some specific lesson ideas for using calculators in the classroom. Section Five focuses on programmable calculators. Section Six contains articles which consider ways in which microcomputers can be introduced into schools, addressing physical, economic, and political issues. Section Seven explores implications of the computer on mathematics curricula, considering both new topics and new approaches to old topics (such as computer assisted instruction). Computer literacy is the theme of Section Eight, suggesting that although all students need to know about computers, "what" they need to know is debatable. The ability to simulate real-world events (computer simulations) is considered in the final section, suggesting that this ability opens new areas for mathematical exploration. (Author/JN)

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CALCULATORS, COMPUTERS, AND CLASSROOMS

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CALCULATORS, COMPUTERS, AND CLASSROOMS

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CALCULATORS, COMPUTERS, AND CLASSROOMS

Introduction

The developers of electronic chip technology may have dreamed about revolutionizing the industrial world, but it is doubtful that they realized the extent to which this technology would revolutionize the educational world. Nowhere does that revolution have more potential for impact than in mathematics classrooms from kindergarten to college. Paradoxically, these silicon-based chips free us from routine thinking and stretch our minds to new thinking patterns at the same time. This collection of readings explores that paradox and points the way to the mathematics curriculum of the future.

The book begins by considering simple hand-held calculators, proceeds to investigate programmable calculators, and concludes with an examination of the educational implications of microcomputers. The distinction between these three is often fuzzy and hard to discern. The reader should not be overly concerned about forming fixed categories. Microcomputers do basic four-function arithmetic just as hand-held calculators do. But both programmable calculators and microcomputers do more than calculators. Calculators add, subtract, multiply, and divide. They may raise to powers, take roots, find logarithms, and calculate trigonometric functions. To these feats, the programmable calculator adds one more: it can test an equality or inequality to determine if is true or false. This is the basic step for logical programming and decision-making. To this extent, the programmable calculator is essentially a computer. But a microcomputer usually can do one additional thing: it can operate with a programming language, using numbers, letters, and/or symbols as commands as well as elements to operate on.

If there is a logical progression between these machines, there is probably a logical progression (in a similar manner) in the way the machines may be used in classrooms. The calculator can free students from routine computation and enable them to focus upon results and relationships. The programmable calculator adds the aspect of logical testing and sequencing to the realm of study. And finally, the microcomputer expands our capabilities to give instructions and to study the instruction-giving process. Ironically, this is a return to the idea of algorithm -- but algorithm on a thinking, analytical level rather than on a routine, mechanical level. Thus, the silicon chip both frees us from thinking and stretches our thinking requirements at the same time.

The first section of this book considers general implications for the mathematics curriculum implied by this calculator freedom. It is balanced by Section Two exploring inappropriate ways in which calculators can be used. Freedom from thinking about routine calculations provides freedom for thinking about problem-solving, the theme of Section Three. Section Four concludes the consideration of calculators by including some specific lesson ideas for using calculators in classrooms.

Section Five considers programmable calculators. Just as the programmable calculator is a bridge between calculators and computers, this section is a bridge between the initial and final sections of the book.

Section Six considers ways in which computers can be introduced into schools. As such, it is concerned with physical, economic, and even political considerations. The following section (Section Seven) explores implications of the computer for the mathematics curriculum, and considers not only new topics, but new approaches to old topics as well. Computer literacy is the theme of Section Eight. It is generally agreed that all students need to know about computers; it is not generally agreed just what they need to know in order to function in tommorrow's society. This section explores questions of "need to know" and "nice to know." The book concludes with a hint of the possible range of the use of computers: the ability to simulate real-world events. This ability to mimic change and the change-process opens new areas for mathematical exploration and makes them accessible to all classrooms.

This book is a glimpse at the future. Like a quick glance, it may be incomplete and perhaps even distorted in parts. But we think it is an exciting glimpse, and we hope you will share that excitement with us.

Jon L. Higgins

Vicky Kirschner

Editors

Section I

CALCULATORS

and

MATHEMATICS

CURRICULUM

CALCULATORS IN THE CLASSROOM:
A PROPOSAL FOR CURRICULAR CHANGE*

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*Paper presented as part of a Symposium "The Effects of Calculator Availability on School Mathematics Curriculum" at the Annual Meeting of the American Educational Research Association, San Francisco, April 9, 1979.

CALCULATORS IN THE CLASSROOM:
A PROPOSAL FOR CURRICULAR CHANGE*

Historical Perspective

A careful study of the history of mathematics education will reveal that computation has always been the focus of the elementary school mathematics curriculum. In the 18th century, children were taught ciphering. Rote computation was taught without any attempt to develop an understanding of the process. During the 19th century a few persons, like Warren Coburn, called for attention to meaning but the curriculum remained computational. During the 1930s there was a movement toward social utility and developing meaning in mathematics. Then in the period from 1958 to 1971 there was an emphasis on teaching the structure of mathematics. Viewed from the perspective of today, there was one unfortunate aspect of the so called "modern mathematics" movement. Much attention was given to rationalizing algorithms. Each of the complex algorithms (e.g., the division algorithm) was taught in great detail using a subtracting approach so that students would understand why the algorithm worked. But computational algorithms are nothing more than a set of rules for efficient paper and pencil answer-finding.

While it is important for students to understand the mathematics they are learning, time spent teaching children why algorithms work does little to help pupils understand mathematics and its applications. Vestiges of this unfortunate curriculum trend remain in current textbooks. Since 1971 there has been a well-defined swing toward the "Basics." In elementary school classrooms, this has been interpreted as more emphasis and time teaching basic facts and computation with even less time for concepts and applications.

Throughout the eras described above, the curriculum has remained computational. There is great contrast between the recommendations of leading mathematics educators and practices in the classroom.

At the same time this country was experiencing a Back-to-Basics movement in the 70s, advances in electronic technology were reshaping the mathematical needs of society. Today the availability of inexpensive calculators has eliminated the need for complex computational algorithms. Before calculators (B.C.) it was necessary to be proficient in computations to apply mathematics. That has now changed. Cash registers show the amount of change and nearly every home and office has a calculator which can perform complex computations rapidly. It is quite ironic that as the calculator availability was reducing the need for computation, schools were increasing their emphasis on paper and pencil computation proficiency.

Current Status of the Elementary School Mathematics Curriculum

A study of currently available elementary school mathematics texts suggests that the curriculum is computationally oriented. A typical fifth grade text devotes 139 pages (37%) to computation. However, teachers do not teach all the chapters. For a number of reasons decisions are made to omit certain chapters. Most classes cannot finish the text in the time alloted for mathematics. In practice, the chapters omitted are usually noncomputational. Chapters covering geometry, measurement, probability, statistics, and problem-solving are first targets for omission. The highest priority is given to teaching computation. In fact, while certain topics are omitted, daily work is supplemented with additional computational worksheets. In many classes, computation completely dominates the year's work. My experience and that of my colleagues observing in schools almost daily for an entire school year suggests that computation is the major emphasis in nearly every classroom. Calls for accountability and minimal competency testing have increased the time spent teaching computation since parents and school boards want to be sure that children know their "Basics." The newer texts reflect this trend.

The Proposal

1. Shift from a computation-based curriculum to a conceptually oriented curriculum utilizing the calculator as an instructional tool.

2. Eliminate the teaching of complex computations in the elementary school.

Rationale

1. We cannot afford the cost in time of teaching complex computations. Through surveying teachers, observing, and analyzing textbooks, I have concluded that of the first nine school years in studying mathematics more than two years are devoted to teaching the division algorithm. When the level of proficiency attained by the pupils is considered, along with this information, I conclude that we can no longer afford to teach complex long division. An analysis of other computational procedures will reveal that the cost benefit ratio is far too high. The time saved by dropping this topic could be devoted to applications and problem-solving where the focus would be on when to divide, not how to divide.

2. Complex computations by paper and pencil are no longer necessary.

3. A computationally oriented curriculum inhibits the development of problem solving. A heavy emphasis on algorithmic thinking tends to encourage the application of rule-seeking behavior, even when it is not appropriate. Successful problem-solvers need to try a variety of approaches, apply heuristics, and recognize that more than rule identification is required. With a calculator to perform computations, pupils are freed to focus on the problem rather than being distracted by switching to algorithmic thinking to compute.

4. The NAEP data suggest that 13-year-olds are not proficient in complex computations, even with all the time devoted. An examination of the National Assessment data (Carpenter et al., 1978) reveals that less than two-thirds of the 13-year-olds could perform a long division problem (three digit by two digit). Only 45% of the 13-year-olds could solve a single-step word problem requiring single digit division.

5. Other topics are of more importance. In order to use mathematics meaningfully in today's society an understanding of certain other topics is important. More attention should be given to estimation, interpreting data, geometry, measurement, and the use of decimals and percents. Time will be available for these important topics only if computation is deemphasized.

Recommendations for Curricular Change

Specifically, I am proposing that the following complex computations be excised from the curriculum.

1. Division with two or more digits in the divisor (e.g., $37\overline{)296}$).

   This topic consumes more time than any other single computational topic in the elementary school curriculum. On the other hand, the single digit divisor algorithm is relatively easy to teach and could be retained. It is critically important that pupils learn the meaning of division. This can certainly be accomplished with single digit divisor computation. While I am recommending that calculators be used to perform complex divisions, attention should be given to interpreting quotients and estimating results. It is essential that children attach meaning to their calculator results. Calculator activities have been published which focus on this objective (Vervoot and Mason, 1977, and Reys, et al., 1979a&b).

2. Multiplication by two or more digits (e.g., $\begin{array}{r} 462 \\ \underline{\times\ 89} \end{array}$). Children should gain proficiency with problems such as

$$\begin{array}{r} 326 \\ \underline{\times\ 8} \end{array} \quad \text{and} \quad \begin{array}{r} 39000 \\ \underline{\times\ \quad 4} \end{array} .$$

3. Addition of fractional numbers with unlike denominators. This topic has proven to be one the most difficult and thus time-consuming topics in the curriculum. The difficulty may result from the cognitive demands of the task. Addition of fractional numbers requires formal thought and most elementary school pupils have not reached this cognitive level. This skill can probably be taught more efficiently if begun in the junior high school.

4. Complex computations with decimals. Such tasks as 3.45 x .865 or 456.78 ÷ 6.7 are more appropriately performed on a calculator.

What Should We Teach?

I will not attempt to describe a new curriculum in detail but instead make certain clarifying points related to the proposal.

1. Insist on mastery of basic facts.

   It is important that children memorize the addition and multiplication facts. As Professor Schoen has shown, the calculator can be quite useful in teaching the basic facts.

2. Stress the concepts of addition, subtraction, multiplication, and division. It will be easier to do this if less emphasis is placed on computation.

3. Stres estimation and mental arithmetic.

   With calculators used for complex computations there is much importance in children being able to estimate the answer, and to judge whether the result makes sense. Along with this, more time teaching mental arithmetic would be quite useful.

4. Emphasize applications.

   Mathematics is studied in large part because of its utility, yet the curriculum has not included much attention to applications for two reasons. We have reasoned that pupils need to learn to compute before they can apply mathematics and time has not been available because of the emphasis on computation. This must change.

5. Emphasize decimals.

6. Be sure that such topics as measurement, statistics, probability, and problem-solving are taught.

   These topics have appeared in elementary school texts but have been given low priority. They should be considered as basic.

7. Include computer literacy.

   Since computers play such a central role in today's society, children should begin to learn about them at an early age.

The NCSM list of 10 basics (1977) should become the foundation for curriculum development. Number five on the NCSM list is "appropriate computational skill." This paper suggests one interpretation of this statement.

The Role of the Calculator

Although this proposal seems to stress the calculator as a substitute for paper and pencil computation, the calculator is actually a highly versatile instructional aid. A calculator can be used in concept development quite effectively since many examples can be explored quickly. Consider the concept of prime number. A number can be tested for primeness quickly by dividing on a calculator. Pupils must know what divisors to try but the focus can remain on prime since the child does not have to take the time to perform the paper and pencil division. The concept of decimals can be introduced quite early and developed through calculator activities. More realistic numbers can be used since the calculator handles "grubby" numbers just as well as "nice" numbers. Many motivational calculator games develop important concepts such as place value, estimation, integers, and functional rules.

Summary

The curriculum changes I am proposing may seem radical but the mathematics needs of society have changed in the last few years with the advent of electronic technology and will change even more in the near future. A computationally oriented curriculum is archaic and will not prepare our students for the 21st century. We must shift the emphasis to applications and topics that now are crowded out by the time spent on computation. Only by excising complex computations can time be made available for applications, problem-solving, and other important topics. Such changes have far reaching implications, from teacher education to textbook preparation.

This document may be obtained from EDRS as ED 175 631.

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Towards a Definition of Basic Numeracy

MICHAEL GIRLING

At a time when a cheap calculator can be bought for the price of two good cabbages, we need to redefine our aims for numeracy.

I suggest the following definition.

Basic numeracy is the ability to use a four-function electronic calculator *sensibly*

If this definition is accepted it is obviously necessary to re-examine our objectives in the teaching of calculation in mathematics, particularly where it is related to the paper and pencil algorithms which form the core of this work in primary schools.

The definition needs tightening up by expanding what I mean by "sensibly". This seems to me to be the crucial question, and I will try to define "sensibly" generally and by giving specific examples. I will then consider the place of algorithms in the scheme of our approach to mathematical calculation.

1 We need to be able to *check* that we or the calculator have not made a mistake and given a wrong result. This checking can take several forms and, depending on the seriousness of our concern in getting the correct answer (to the necessary accuracy), we may use one or more of them.

- a) Does the answer make sense? (323·63 miles is a long walk.)
- b) Repeat the calculation in a different order, using, if possible, a different operation, at least three times.
- c) Very rough approximation. (Is the decimal point in the right place?)
- d) Approximation to one figure accuracy (in terms of salaries, say, £3,000 is very different from £1,000 or £9,000).
- e) Use of pattern. (32×87 should end in 4; 3002×9007 probably won't though!)
- f) Work out intermediate results and use the previous checks on them. (Mr. Hope-Jones, when he was a master at Eton, was presented with a rubber stamp by one of his sets which said, "You have substituted for π too early". This stamp is now obsolete!)
- g) It is important to check that the input data is reasonable. (9,291 boxes of chocolates at 3p each?)

2 We need an understanding of the relative size of numbers. What numbers are appropriate to describe the number of pages in a book or the number of words on a page, say. How long is a million seconds; how short is a millionth of a second? This skill is not easily acquired. We would make a start if every calculation we asked for, except for those needed to investigate pattern and structure in the number system, were related to a realistic problem. We should *never* require a calculation for which "Does the answer make sense?" is an irrelevant check.

3 We need to be able to perform *mental* calculations for speed, for convenience and so as to be able to hold our own in the commercial and industrial world. The standard of mental skill we achieve will vary, and depend on our ability and interest. The minimum should probably contain:

- a) the ability to give and receive change by "counting on";
- b) multiplying and dividing by 10;
- c) addition facts to 20 (quick recall);
- d) doubling and halving (varying degrees of difficulty);
- e) multiplication facts to 10×10 (abolish table squares!).

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The standard will vary mainly in the ability to chain these processes together and in the speed with which they are performed. Speed should always be considered less important than accuracy. New strategies need to be devised to encourage and promote mental work, perhaps a new campaign: "*Dont* show your working"!!

Algorithms

I am not going to suggest that pencil and paper algorithms should not be taught, but that they should *only* be taught as part of the armoury of techniques that we have to help in an understanding of number *and not because they are useful.*

This is not just a slight difference in emphasis but, I believe, has quite dramatic consequences:–

a) The concentration is now unequivocally on understanding the process involved.
b) There is a very clear advantage in studying as many different algorithms as can be manufactured; for example *all* possible methods of subtraction should be used!
c) The most refined methods of long division, for instance, which may be the least illuminating, need not be taught—at least not to everyone.
d) There is no need for anyone to be stopped in their progress in mathematics through being unable to perform the useless algorithms we now require.

It is perhaps worth adding that the time that might be saved by cutting out the practice of techniques could be well used by more attention to

a) increasing mental facility;
b) early introduction and manipulation of scientific notation;
c) developing techniques of approximation;
d) serious investigation of pattern in number.

(Mr. Girling wishes it to be known that his views are understood to be controversial, and are not necessarily shared by his colleagues. The implications need to be discussed not just by teachers but by employers, parents and teachers in tertiary education.)

Section II

CALCULATOR

CAUTIONS

Calculators in the Elementary Classroom: How Can We Go Wrong!

By Robert E. Reys

Hand-held calculators are widely available. The 1977–78 National Assessment of Educational Progress reports that 75 percent of 9-year-olds, 80 percent of 13-year-olds, and 85 percent of 17-year-olds have access to at least one calculator. State and local surveys document that this statistic increases yearly.

The current proliferation of calculator-related activities for school programs are evidence of the impact of hand-held calculators. Dramatic changes have been reflected in basal series and supplementary publications that are being designed to take advantage of the existence of calculators. Thus the role of calculators in elementary school mathematics must be seriously considered and raises a pertinent question, namely, How can we go wrong?

We can go wrong by—

1. Banning the use of calculators in mathematics classes

A tremendous amount of inertia is to be overcome if calculators are to have an impact on elementary school mathematics programs. Several things contribute to this inertia.

(a) Few calculators exist in schools. Despite the fact that most children either own or have access to a hand-held calculator, only slightly over 20 percent of schools reported having calculators (Wyatt et al. 1979).

(b) Basal textbook series have yet to integrate calculators into their programs as an instructional tool. Only 11 percent of the teachers reported that their mathematics textbooks included activities written for calculator use. Interestingly enough, a majority of teachers thought their texts should include activities that use calculators (Reys et al. 1980). Furthermore, few supplementary materials exist that provide pragmatic use of calculators with content typical of today's mathematics program.

(c) Lack of familiarity with calculators. Only 4 percent of the elementary teachers had attended a calculator workshop, yet more than two-thirds of them wanted to learn ways of using calculators in their mathematics classes. (Reys et al. 1980).

This documents the need for a massive transfusion designed to provide awareness and effective instructional uses of calculators in elementary schools.

2. Forbidding the use of calculators on standardized tests

More than 80 percent of the elemen-

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tary teachers said calculators should be available to children in school. (Reys et al.) This is a strong endorsement of calculator use, yet nearly all of the teachers cited standardized tests as the principal reason for not using calculators. Teachers feel responsible for preparing students for these tests and since computation is a separate strand in almost every standardized test, emphasis is placed on paper-and-pencil computation. The second mathematics assessment by the National Assessment of Educational Progress reports that students using calculators do better on direct computation, but the use of calculators has little effect on problem solving. (Carpenter et al. 1980)

Suppose your school uses calculators the next time standardized tests are administered. What would likely happen? There will be a jump in computation scores, with little change on other portions of the test. Foul! Foul! you cry. This isn't fair—children aren't supposed to use calculators on tests. Why? The following problem is one selected from the mathematics problem-solving portion of a widely used standardized text and slightly modified:

> In servicing a car the attendant used 5 quarts of oil at 75¢ a quart and 15 gallons of gasoline at 86¢ a gallon. What was the total cost for oil and gas?

If a child were allowed to use a calculator to find the answer, would the calculator do the problem for the child? Does the calculator decide what keys to punch? Does the calculator interpret the result? Calculators don't think for you; they only do what they are told to do. Many employers recognize this and ask prospective employees to bring a calculator to use on job application tests.

Test developers are now preparing forms with which students can use calculators. If a few schools allowed students to use calculators on standardized tests, it would have a tremendous impact on test development and accelerate the construction of assessments using calculators. Calculator forms of standardized tests would soon appear. Most importantly, when teachers know calculators can be used in all stages of the learning process—from introductory development to evaluation—they can concentrate on problem solving and higher level mathematics learning.

3. Failing to recognize the intangibles associated with calculator use.

Dramatic increases in computation performance are consistently associated with calculator use. Less objective, yet extremely strong evidence suggests that pupils show more enthusiasm toward and confidence in solving problems; greater motivation for learning, which is accompanied by a more positive attitude toward mathematics; greater persistence in solving problems, which results in more time on the task; recognition of different calculator solutions to the same problem; and increased awareness of weaknesses and limitations of calculators, such as calculators performing only what the operator keys in, overloading, stroking errors, or mechanical errors that sometimes occur in the calculator. (Wheatley et al. 1979) All of these are extremely valuable educational goals and should be considered in evaluating the impact of calculators in the classroom.

4. Creating another false dichotomy

In recent years, mathematics education has experienced a number of false dichotomies such as new mathematics versus old mathematics, basic skills versus regular mathematics programs, skill versus concept development, and discovery versus expository lessons.

To consider calculator programs versus noncalculator programs is to create another false dichotomy in mathematics education. Calculators must be viewed as another tool within the mathematics program and used as such. There are many topics appropriate for calculator usage and others equally inappropriate. These differences must be recognized and treated accordingly in developing instructional programs. Thus, the question is not should calculators be used, but when, where, and how can they be used most effectively.

5. Using calculators to check paper-and-pencil calculations.

Teachers often cite checking paper-and-pencil calculations as the best use to be made of calculators by their students. Over 85 percent of the elementary teachers indicated that calculators should be used to check paper-and-pencil computation. (Reys et al. 1980) This is perhaps the most popular use of calculators by teachers in elementary schools today. Yet *asking pupils to do lengthy calculations and then asking them to check their work with a calculator is of questionable educational value.* Providing an easily accessible answer key is a much more efficient use of a child's time. Furthermore, children wonder why they should spend so much time doing the paper-and-pencil calculation if a calculator is available. This reinforces the notion that use of calculators is cheating, a feeling that should be avoided under any circumstances. *If calculators are to be used to check work, it is far more realistic to perform the calculation twice using the calculator and to compare these answers.*

6. Emphasizing the calculator in developing paper-and-pencil computation algorithms.

It has been suggested that paper-and-pencil computation algorithms be developed with the calculator. For example, the calculator could be used to find the partial products in figure 1, but why would you want to? This is a very artificial use of a calculator. If you have a calculator available, it is much more reasonable to do the computation with a single operation.

There are, of course, calculator algorithms which are important and require explicit development. For example,

$$\begin{array}{r} 1234567 \\ \underline{\times\ 7007} \end{array}$$

presents an interesting problem that cannot be solved directly with most calculators. Yet thoughtful use of a calculator and a modification of the typical multiplication algorithm provides a speedy and accurate solution. In fact, the most appropriate algorithm for many computational problems will depend on the quantities and the internal logic of the calculator. Our teaching must be designed to help students decide which algorithm or combination of algorithms is needed to solve a particular problem.

Fig. 1

$$\begin{array}{r} 463 \\ \underline{\times\ 24} \end{array}$$

$4 \times 463 \longrightarrow \boxed{}$

$20 \times 463 \longrightarrow \boxed{}$

7. Using calculators only after basic facts have been mastered.

Many exciting ways of using calculators to help develop basic facts exist. Research evidence indicates that the development of basic facts is, in fact, enhanced in the primary grades through calculator use. (Channel 1980). To unequivocally ban calculators or ignore their potential contribution in the development and mastery of basic facts is a serious error of judgment.

8. Using calculators only after the concept has been taught.

Calculators can be used to reinforce and expand many mathematics concepts that have been introduced. No research evidence exists to support the claim that concepts must be developed prior to calculator use. (Suydam, 1979). The notion of developing understanding through examples, followed by explanation and discussion is a common technique in mathematics teaching. Why shouldn't a calculator be used to give pupils many examples of concepts. such as negative numbers, decimals, or exponents before, or at least in conjunction with formal instruction on these topics? This question deserves careful discussion by teachers and attention by researchers. In the meantime, the use of calculators will be used in all learning stages and not arbitrarily restricted to concepts that have already been taught.

9. Assume that concrete materials or manipulatives are less important in the learning process.

With the increased computational facility among young pupils, it is tempting to bypass models (such as counters, an abacus and multi-base blocks) in developing place value concepts and/or computational algorithms. Omission of, or even less attention to such models would be dangerous and increase the likelihood of mechanical rather than meaningful learning. The challenge is to plan concrete models which parallel and complement the calculator experience.

10. Assuming that the process of formulating large numbers is accelerated.

Calculators certainly provide many opportunities for experiences with small and large numbers. Facility of displaying or reading numbers may be falsely interpreted as understanding. Research has shown that understanding the formulation of numbers is a delicate cognitive process and that it is developed over a long period of time. There is no research evidence to suggest that this period of concept development will be shortened or that any stages will be bypassed through calculator experiences.

Where to from Here?

Much research related to the use of hand-held calculators in elementary schools has been completed. Without doubt the most important result from this research has been its consistent findings that "almost all of the studies comparing achievement of groups using or not using calculators favor the calculator groups or reflect no significant differences." (Suydam 1979). This suggests that schools that have calculators have everything to gain in using them as an instructional tool in their mathematics program. At this stage, the most clearly needed research is longitudinal—at least several years in length—studies of students who have had sustained school experience with calculators. In the meantime, we cannot ignore calculators and simply hide our heads in the sand.

As with any new technological development, there are many things that will change. As a result, there are many places to go—some right, others wrong. The only clear wrong decision at this time would be for teachers and schools to ignore or ban hand calculators, thus not considering them in future development of their mathematics programs. It is clear that hand calculators hold great potential as an instructional tool for the elementary school. The educational value has been proclaimed by many, including the National Council of Teachers of Mathematics which "encourages the use of calculators in the classroom as instruction aids and instructional tools."

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Calculators: Abuses and Uses

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A look at much of the calculator material currently available for use in classrooms suggests that we are in the midst of a fast growing technological movement, one which has not allowed us much time to consider how best to implement this technology. Calculator books and magazines are in abundance, and much of what appears in these supplementary materials represents merely play activity, or worse forces the use of the machine with little attention to the goals of school mathematics. It is possible to classify the most common abuses (abuses in terms of the activities used as a part of school mathematics, not necessarily in terms of their potential interest to individuals) into four general categories.

1. **Calculations,** often with awkward numbers, *for no apparent purpose* other than to require the use of the calculator. Some of the proposed calculations might be interesting or motivating to some because of the chance to use the calculator, but a full page of 'big number' calculations looks formidable whether or not the exercises are to be done with a calculator. One can raise the question regarding this type of abuse: Will such activity create different attitudes towards mathematics? Pupils will still make mistakes pressing the buttons, and when practising in such situations may well be inclined to say "So what". It is possible to use some of the big number calculations which are embedded in so-called applications to develop some sense of large numbers, but because of the crude measurements and approximations which must be used, most of these activities are more appropriately estimation exercises rather than calculator exercises. As an example, take the activity about the length of the line if all cars in the U.S. were lined up bumper to bumper, which appeared in the article *Calculations You Would Never Make Without a Minicalculator* [1]; one might suggest that the article should have been titled *Calculations You Would Never Make.*

2. **Games and puzzles** with no apparent mathematical objectives. This is not meant to suggest that games and puzzles are not a valuable teaching aid (in fact, the November 1976 *Arithmetic Teacher* describes a number of such activities which fit very nicely into a school mathematics curriculum, and a great number of the calculator games have a logical reasoning component—an important goal in school mathematics), but rather it is intended as a caution against an indiscriminate use. So the listing of this as a category of abuses is not intended to suggest that learning should not be fun, but rather that such fun should be used to advance our cause. This is particularly important since an often heard teacher complaint is that the curriculum is already so full that they are hard pressed to "cover the existing material".

3. **Mystical button pushing.** One of the most frequent abuses, and a good example for this category, is 'making words by turning your calculator upside down'—often a fun activity, but one best left for home or pleasure. It represents just one more attempt to coerce children into doing some basic operations with numbers, utilising a calculator.

The button pushing category also includes an even worse abuse; children are asked to use the calculator to perform some activity for which they lack some basic prerequisite understanding. This is *not* an argument for limited keyboard display, as I do not feel there is any problem if a child just wants to see what happens when certain buttons are pushed. Rather, the concern is about requiring responses or procedures for which the child lacks the necessary background for understanding. A good example of this abuse appears in a recent U.S. calculator publication intended for use with primary age children. Early in the first book the children (aged 5 to 6) are asked to use their calculator to count the ones in a display

11111
11111
11

by pressing 1 + each time they count. And this is before any consideration has been given to the concept of addition or the '+' symbol. One can reasonably ask what the author expected to achieve with such an activity—and whether there are not better ways of doing this.

4. **Checking answers.** I expect to find considerable disagreement with listing this category as an abuse. However, why use the calculator to check when it is usually the best device for performing the calculations in the first place? Why not merely give pupils an answer sheet for checking answers; do we need a relatively expensive piece of equipment for this task? (The Editor makes this very point in the December 1977 *MT* [2].) For those who welcome the motivational aspect of the calculator, this may well become 'old-hat' when the curriculum is finally revised to take full advantage of this device in teaching and learning. It seems that with today's technology the more

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critical checking skill is the ability to estimate and assess the reasonableness of answers (more about this later).

One hates to begin an article by listing all the negative points, particularly when the calculator offers so much that is good. However, some caution is called for before we go too far. We do not want to be in a position where a critic questions our use of this device, demanding to know whether it is not just another expensive educational gadget, and we are unable to supply acceptable answers. While it may be true that many of the current educational materials tend to lack any firm philosophical or pedagogical basis other than an emphasis on the motivational aspect of calculator use, it is also true that the majority of these materials do contain a few, sometimes very few, good activities—or at least the basis for some good ones when reworked.

While working with teachers in Minneapolis/St. Paul, I have found it helpful first to establish a scheme for categorising the different types of calculator activities which at the same time relates these activities to the school curriculum. The scheme has been particularly useful for assessing the potential contribution of published materials and identifying areas which need development. (For an alternative list of categories, see [3].) The placement of a particular calculator lesson in one or more of the categories is based on the *purpose* of the activity. The categories are not intended to be distinct or to represent disjoint sets, but they merely provide a structure for thinking about calculator activities: one should not get hung up on whether a particular activity belongs, say, under *patterns* or *exploration*. A consideration of these categories should also enable a teacher to provide some balance in types of use. The category *new/renewed content* is included to provide an opportunity to consider curricular changes or the renewed emphasis which might be expected in school mathematics because of the availability of calculators and computers.

The examples below come from a variety of areas and are only intended to be illustrative of the types of activities in each category.

Calculations

This is really the most obvious category, and it is usually easier to describe it last after showing examples for each of the others. It is often the case that mathematical ideas and procedures can be best illustrated with small, easily handled, numbers—hence it may not be necessary or even desirable to change this just to make use of a calculator. It is important that we identify situations where the contribution is 'real' rather than forced.

One example, for which the calculator's role is *primarily* for doing the arithmetic, is in working with the formula for the number of combinations of n things taken r at a time. We are often forced to leave a result in notational form, e.g.:

the number of possible poker hands:

$$^{52}C_5 = \frac{52!}{5!(52-5)!};$$

the number of possible bridge hands:

$$^{52}C_{13} = \frac{52!}{13!(52-13)!}.$$

This may be pretty unsatisfying—and pupils may have little feel for the relative size of these numbers. Do you? Guess which of the above you think is larger (and how much larger) and then check with your calculator. The calculator does not remove the need to be able to work with factorials and to be able to represent the expansion in reduced form, as 52! would involve a lot of button pushing and may overflow the machine. However, the calculator does remove the computational drudgery involved in multiplying many numbers. (The answers to the two exercises are 2,598,960 and 635,013,559,600 respectively, and the second is considerably larger. Does this surprise you?)

Another example involves the solution of trigonometric equations. If the equation has the unknown in the denominator, e.g.,

$$\tan 63° = 22/x,$$

solving for x requires division by a four or five place decimal, usually taken from a table (or with a shrewd manoeuvre, multiplication by a four or five place decimal). Since the emphasis is really on setting up the equation and using the appropriate trigonometrical function, the calculations invariably end up getting in the way of the straightforward ideas. Examples from trigonometry really require little discussion as it is readily apparent that for such things as building a table for sketching a reasonably accurate picture of the graph of the sine or cosine functions, and for applying the sine or cosine law, the calculator can be an invaluable aid. The availability of the calculator also suggests that table-reading activities generally included in certain mathematics courses can be considerably reduced.

An entry under *calculations* has the characteristic that its primary purpose is to remove computational drudgery and the actual input or output is *not* intended to demonstrate or reinforce concepts or have a strong application or problem-solving emphasis. (In the latter cases the activity falls into one of the other categories.) The need for including this category will become more apparent when the examples for the others have been presented. It will also become apparent that the categories are in fact somewhat hierachial, as it is generally the case that the calculational aspect of calculator use will generally be a secondary purpose in activities listed elsewhere. Remember that the key element in determining where a particular activity or lesson fits in the scheme is in terms of the *primary purpose of the lesson*.

Patterns

This may well be considered a part of *renewed content* or a type of *exploration*. However, since it is considered to be an important mathematical activity in its own right and a useful skill for solving certain types of problems and for learning new mathematics, it is listed as a separate item. In this way it does not get slighted or lost. Many of the articles and booklets which promote the use of calculators make a big point of emphasising this activity; however, if one looks carefully at the situations which are described, one finds that typically they tend to emphasise the generation of easily observed patterns, as opposed to generating output which then requires some study and testing to find a pattern. Thus, one might wish to divide pattern activities into *pattern generation* and *pattern search*. Both have a role, but unfortunately the first is often left without taking full advantage of the setting. Consider the following, quite common, example.

Complete the following.

$37 \times 1 \times 3 =$
$37 \times 2 \times 3 =$
$37 \times 3 \times 3 =$
$37 \times 4 \times 3 =$

Is there a pattern? Use the pattern to complete the rest of chart. (And the children are asked to find the results up to $37 \times 9 \times 3$.)

While this might be interesting, the situation as given does not go anywhere. The pattern is obvious and one merely writes down the triples. Children at an early level catch on very quickly and complete the items without much thought. When the patterns are of such an obvious nature, one needs to ask how the situation can be used to promote mathematical thinking. One natural extension is to ask the children to try to figure out why it works. In the example the key is to notice that $37 \times 3 = 111$. (One can extend this same example to more of a *pattern search* activity by asking the child to try to figure out the pattern up to $37 \times 60 \times 3$. In this case they will have to try to figure out how the pattern seems to change in each decade, and at what point. A pupil worksheet and accompanying teacher notes for the extended calculator activity was produced by a team of Minneapolis teachers and included in a set of materials called *Calculator Cookery*. [4])

If the pattern lessons are to be primarily *pattern search*, then the pupil should be involved in a searching and testing situation, that is he should be required to study the various examples, to hypothesise a relationship, to test or verify the relationship, to modify on the basis of new information, etc. Examples of this type of activity abound in the literature, but are probably not as prevalant as the first type. Here are three examples which illustrate the type of problem-solving thinking which must be employed to find the pattern.

1. *Use the calculator to find the recurring cycle of digits for 1/7, 2/7, 3/7, . . . 6/7 (and 7/7). What do you observe? Try the set of fractions with denominator 13. Again, what do you observe? The decimal expansions for some fractions like the 7ths are cyclic in the sense that the repeating digits appear in the same sequence, but with a different initial digit. The set of fractions with denominator 19 are also cyclic. Knowing this use your calculator to find the decimal expansion for 1/19.*

(For a discussion of methods see several *MT* articles culminating in [5].)

2. *Generalise:* $1^3 =$
$1^3 + 2^3 =$
$1^3 + 2^3 + 3^3 =$
. . .
$1^3 + 2^3 + 3^3 + \dots + n^3 =$
Use your calculator to test your generalisation for $n=8$, $n=10$.

3. *The question to be investigated is: If a is assigned an integer value such that a is odd and greater than 1, is it always possible to determine integers b and c so that* $a^2 + b^2 = c^2$*? Use the following examples.*

1.	$3^2 + b^2 = c^2$	1.	$5^2 + b^2 = c^2$
2.	$3^2 + 4^2 = 5^2$	2.	$5^2 + 12^2 = 13^2$
3.	$9 + 4^2 = 5^2$	3.	$25 + 12^2 = 13^2$
4.	$9 + 16 = 25$	4.	$25 + 144 = 169$

Try these.

1.	$7^2 + b^2 = c^2$	1.	$9^2 + b^2 = c^2$
2.		2.	
3.		3.	
4.		4.	

When you find the number write out as the examples.

1. $11^2 + b^2 = c^2$ 1. $13^2 + b^2 = c^2$

Do you see a pattern? If so test your conjecture with these.

$37^2 + b^2 = c^2$ $99^2 + b^2 = c^2$ $151^2 + b^2 = c^2$

In each of these examples, the patterns are not so obvious. While the role of the calculator is still primarily one of *pattern generation*, the major purpose of the activity is the search itself. (Even in the decimal expansion for 1/19 the task is not trivial as the search involves looking at sets of digits, not just the single digit. Try it; the cycle has 18 digits.)

One further point should be made. There are also many *pattern search* type of activities which lead to the discovery or generalisation of a 'mainstream' concept or important and/or useful mathematical relationship. These will generally be listed under the category of *exploration* if they have as a primary purpose that of the learning of the specific piece of mathematics and the *pattern search* activity is of secondary importance. This is illustrated in the first example of the next section.

Exploration

This is probably the most fruitful and wealthiest area for calculator activities, particularly in terms of the usual mathematics curriculum. The main feature of activities which are to be placed in this category is that the pupil uses the calculator to generate output with the purpose that the output will demonstrate a concept or relationship, or that the actual generation of the output will serve to help reinforce a concept which has been taught previously, or that the output will assist in solving a mathematical problem. Thus the category really has three somewhat distinct types of examples.

The example of **exploration for concept-demonstration** is of decimal multiplication. The traditional approach is to show how the multiplication is done by a number of examples in which the decimal fraction is first converted to the standard fraction representation with denominators of ten, hundred, etc. The examples are used to justify the rule regarding the placement of the decimal point (in terms of the number of decimal places in each of the factors). Thus, the usual pedagogical sequence is first to justify the new rule and then actually to implement in practice with decimals. For some reason, some pupils overlook the justification stage and tend to concentrate on memorising the rule, leading to errors later on when practice with decimals involves mixed operations. An alternative approach using the calculator is given below. The pupils are first asked to look for a pattern when the calculator is used to do some calculations the pupils have not yet learned how to do (discover a rule), then justify the result with a few, well-chosen examples, and then practise. The emphasis is on finding the pattern and then asking why it works (very much along the same lines as that recommended as the extension to the common pattern generation activities described in the previous section). Also, in this example, students are asked to concentrate immediately on what is to be learned, rather than first look at a number of seemingly less related examples and wondering where the lesson is going.

Use your calculator to find the products:

$6?\times0{\cdot}2$
$0{\cdot}8\times0{\cdot}6$
$3{\cdot}2\times0{\cdot}8$
$2{\cdot}2\times6{\cdot}4$
$0{\cdot}02\times0{\cdot}34$
$2{\cdot}11\times1{\cdot}22$
$0{\cdot}72\times0{\cdot}6$
$0{\cdot}026\times0{\cdot}003$
...

(Approximately 15–25 items, none with a zero as a trailing digit in one of the factors, or a 5 as a trailing digit when the other factor has an even number as the trailing digit.)

What do you observe about the placement of the decimal point in the answers?

(Now the class can discuss any observations and come to the generalisation. The teacher now asks why this is so, and the justification should be readily apparent from a few examples with fractions which illustrate that tenths times tenths gives hundredths, tenths times hundredths gives thousandths, etc.)

Now try the following with your calculator:

$2{\cdot}44\times0{\cdot}35$
$126\times0{\cdot}45$
$3{\cdot}60\times0{\cdot}40$

(5–10 items which involve zero or 5 as a trailing digit)

Does your rule still work? What's wrong?

(This is used to help reinforce the idea that the rule still holds as the calculator suppresses the trailing zeros in the result. Also this reinforces the idea that two-tenths is twenty-hundredths.)

When a particular piece of mathematical content has already been introduced and the calculator activity is planned to provide an opportunity to practise or apply what has been learned, this can be thought of as **exploration for concept-reinforcement**. An example of this type of use can be taken from elementary algebra and involves the evaluation of a^n for various non-negative values of a, taking n to have integer values from say 1 to 10 or 1 to 20. This can be given after the pupils have been introduced to the idea of what is meant by an exponent, and used to reinforce the idea that the exponent says how many times a particular number is to be used as a factor. If pupils are told to consider $a=0$, $0<a<1$, $a=1$, and $a>1$, then some important relationships are also reinforced (or demonstrated if the ideas are new); in particular the fact that for $0<a<1$, a^n gets very small quite quickly, even when a is close to 1, say 0·9. This reinforces (or calls to one's attention) the fact that when one multiplies two proper fractions, the result is always less than the smallest fraction. The calculator use in this lesson illustrates the *dynamic* nature of the result—one of the real advantages of using calculators. Of course, the same example can be used to illustrate how quickly the result grows for numbers greater than 1. (Compare say $a=4$ with $a=8$; the successive terms for the second are not just twice the first.) Thus the activity can be used to provide an interesting first exposure to the exponential function.

Another example of exploration for concept-reinforcement is the calculator game Wipeout (described in the *Arithmetic Teacher*, November 1976, p. 516). This is included here to illustrate that there are some calculator games which have the potential for a very real contribution to the learning of mathematics. The game involves entering a given number, with all digits different, and then asking the pupils to use subtraction to remove a specified digit (i.e. replace it with a zero) without changing any other digit in the number. For example, if the game is used to reinforce concepts of place value, one might start with something like 834·56219 and ask pupils first to remove the 2, then the 4, and so on. (This reinforces place value, since to remove the 2 requires the subtraction of ·002. However, note that this should not represent the only place value practice as it could become a rote activity of the form: count the places, use zeros and subtract.) As indicated previously, there are many games of this type, which can be both motivating and also contribute to the learning of mathematics. However, major consideration should be given to the latter point in deciding whether or not to include such an activity.

Exploration for problem-solving is generally concerned with questions of the form "what happens if", and examples for this category abound in situations which involve formulas. (Formulas are not the only examples, but these easily illustrate the ideas.) One which is both motivating and interesting is to consider the formula (function) which relates temperature in degrees Celsius to degrees Fahrenheit,

$$F=\tfrac{9}{5}C+32.$$

One can ask the question: When, if ever, are the two readings the same? After trying a number of values for C, and looking for a trend or pattern, which is the way one would expect pupils at an *early* level to proceed with the relationship as given, the pupil should finally arrive at the result, $-40°$. (Of course at later levels one would expect pupils to recognise that this problem is readily solved without the calculator by solving an equation.)

Other interesting explorations for problem-solving can be described for such topics as the pythagorean theorem: what happens with different shapes on the sides and hypotenuse; area and volume formulas: what happens if a certain dimension is halved, doubled, tripled, etc.; the classical 1 penny for the first day and double your salary each day for a month; and so on.

Additional explorations can be found in number theory activities as well as many of the classical problems dealing with series and sequences. Examples from this single category could well make up an entire article.

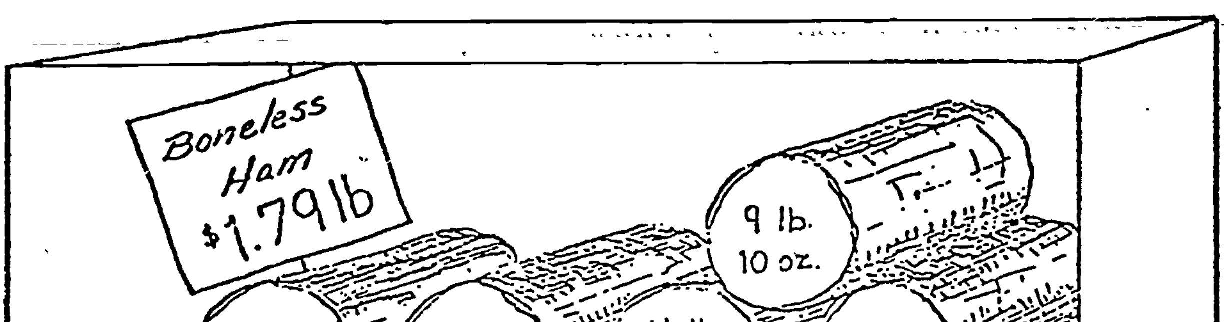
Applications–Consumer

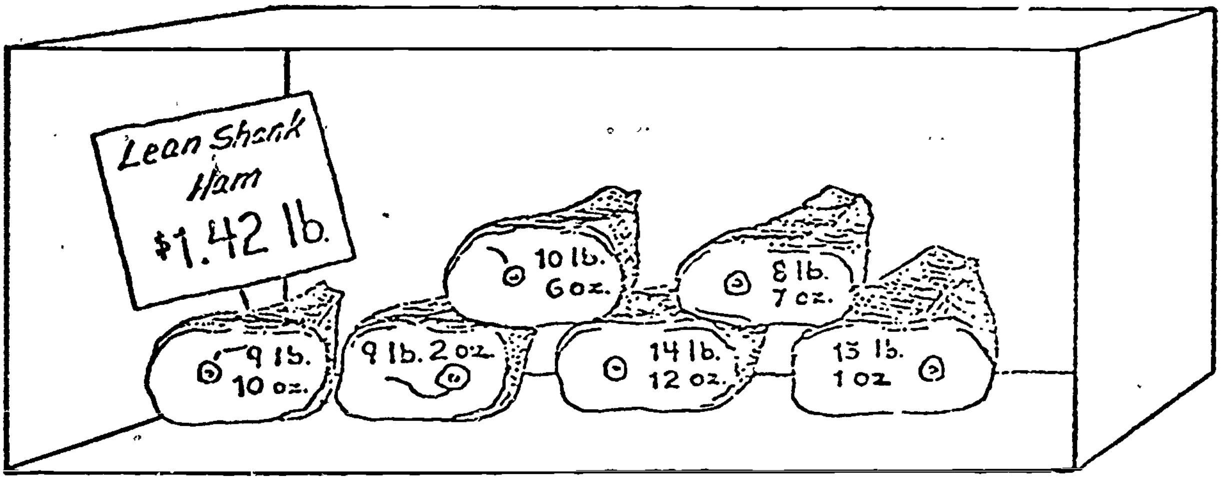
Since the applications area is so important if we are to produce some level of basic numeracy (ability to *use* mathematical ideas and processes), it is useful to separate the applications into two different sub-categories. Consumer applications are those applications which have immediate implications for the individual in everyday living, e.g., comparative shopping; personal consumption of water, gas, and electricity, constructing or making something; etc. Social applications are more oriented to decision-making activities with implications for benefiting society as a whole: population growth; conservation of resources (water, fuel, minerals, etc.); health and health care (e.g., smoking): inflation; and so on.

It is not difficult to find ideas for consumer applications—the real problem is in presenting these in a realistic fashion and in emphasising the role of mathematics in *decision-making*. A nice example of an activity (problem) which is stated much along the lines as it might occur in real life is in the booklet by Dolan [6], p. 38.

PICNIC HAMS

As chairman of the food committee for your club picnic, it is your job to purchase the ham for the sandwiches. The committee plans to make two sandwiches for each person and figures 2 oz. of meat per sandwich. Assume that the bone in the shank ham is 20% of the total weight, and that there are 35 people in the club. Which ham or hams would you buy to make the sandwiches for the least cost?





Another example is to use selected formulas to calculate the cost of running certain electrical appliances (or for lighting). The exercises can be extended actually to figuring monthly bills, using the graduated scale of pricing which is used by the electricity boards, and finding out how one might save by conserving. (What does it cost to leave a heater on all night instead of for only one or two hours in the morning? What would be gained by changing all non-reading lights in your home to 25-watt bulbs?)

Applications–Social

While it would be nice to include two or three examples in this category, one problem is that they generally take some space to develop. Hence I will just describe briefly some of the characteristics of social application lessons. As indicated previously, one key feature of most of such lessons is that the pupil should use the mathematics of a given unit or course to assist in decision-making or to compare alternative solutions. Some lessons may not lend themselves to actual decision-making, but rather enable the pupil to investigate some real world phenomena of an impersonal nature—e.g. develop equations to predict future times for track and field events, look at the manufacturing of some product, etc. However, it is problems of the decision-making type which are sorely lacking in most of the usual textbook applications; the pupils are asked to use their mathematics in what appears at first glance to represent an application, but it goes nowhere or has no apparent purpose. We need to motivate pupils to want to use mathematics and to recognise the value of using mathematics to assist in making decisions. For example, why should one support, or vote against, 'ban the can' legislation (legislation concerning disposable bottles or cans)? What about the use of certain pesticides? One other feature of social applications is that each usually involves either a big problem (for which one needs a reasonable length of time for study) or consists of a number of related exercises, instead of the typical single exercise which culminates in a somewhat open-ended question of a 'why' or 'what' nature. Lessons could be based on such topics as wildlife management, camera lenses, predicting birth rates, sound, etc. (Lessons on these four topics appear in [7].) The main emphasis is on applying mathematical ideas to real world situations. They involve either some aspect of decision-making or the study of some real world phenomena, and any mathematical results need to be interpreted back to the real world. This is quite different from such questions as "How high a pile would we have if all the McDonald hamburgers sold to date were stacked one on top of another?"; at best this is an activity for looking at large numbers (number awareness, *exploration*) and at worst it is merely a button pushing activity which is probably a waste of time.

New/Renewed Content

As indicated earlier, this category is included since it is related to today's technology—and while some of the topics do not necessarily involve the actual use of the calculator on any regular or integral basis, they do in fact relate to the effective use of this technology. The key topics are estimation, errors, algorithms and iteration, and mathematical modelling. (For a discussion of some of these ideas see Michael Girling's *Towards a Definition of Basic Numeracy* [8].) Each of these topics easily warrants an article and it will only be possible to touch briefly on some important ideas here.

Estimation is probably one of the easiest to consider, since it is already a part of today's curriculum. Current practices in many textbooks tend to confuse estimation with rounding off and treat them as if they are the same thing. Even worse, pupils are typically told what to round off to, so they treat this as another mathematical procedure to be implemented when instructed to do so. This results in a failure to identify the procedure as an aid in checking results—and the pupils typically ask what they are supposed to do. With the availability of calculators, the skill of estimation is even more important, but we no longer need the precision in our estimates which was often demanded for checking hand calculation. In fact, to check a calculator result one is primarily concerned with orders of magnitude and hence the only real need is the ability to do single digit arithmetic and work with powers of 10. To estimate the product of, say, 567×6324 (for the purpose of checking a calculator answer) we need only think of this as 500×6000—that is, we need only consider the leading digit (rather than round), a much easier procedure; the product then becomes $5 \times 100 \times 6 \times 1000$, or $5 \times 6 \times 100{,}000$ or 3,000,000. Pupils will soon notice when applying this relatively straightforward algorithm that the resulting estimates are often unsatisfying. This then motivates the need for rounding off. (On the other hand, even if a pupil is unable to round, the leading digit algorithm is still a useful procedure.) After learning to round off, the pupil may still wish to sharpen this skill, and use some number relationship notions and common sense to obtain better estimates. For example, 65×14: using the initial algorithm we estimate 600; using rounding off to the leading digit we estimate 700; using some number sense and thinking about the problem we see we can consider 65×10, or 650, plus 60×4, 240, or a sum of about 900 (and the correct result is 910). Notice that with this three stage approach, the pupil is able to estimate at least the lowest level, and the skill is developed as one feels the need.

The inclusion of the topic of **errors** should probably begin in the primary school. In general this topic can be subdivided into two main categories, one dealing with the concepts of error analysis—the mathematics of errors—and the second related to ideas of checking answers through estimation and on the basis of the reasonableness of the results in terms of physical reality. These concepts are not new, and in the past textbooks have attempted to provide some instruction on the topics. However, it seems that pupils today do not have much of a feel for errors and it would be worthwhile taking another look at what might be done. At the very least we need to include more exercises of the type where an answer is also given and the student is asked to determine whether or not the result is reasonable (and why or why not).

The mathematical notions of *relative* and *absolute* errors should probably be introduced after pupils have some facility with fractions. Pupils should have an idea of the importance of these ideas and how actually to calculate them numerically. That is, the pupil should be able to tell you that dropping the 7 in 0·0067 is a large relative error, while dropping the 7 in 0·6007 is small. (However, we do need to be careful about "large" and "small" as the context of the problem may demand a certain precision.) For the two settings the absolute error, in terms of dropping the 7, is the same. Notions of errors of measurement, precision and accuracy also fit under this topic.

The concept of **algorithm** and the use of **iterative procedures** are basic in today's computer world. More needs to be done with these ideas throughout secondary school mathematics. Pupils need to be given the opportunity to design their own algorithms or to modify existing procedures to do new tasks. In order to develop some appreciation of what an algorithm is, they should be provided with situations which require the following of a new algorithm. One interesting and useful algorithm is bisection, for finding or approximating zeros of polynomial functions; this is definitely within the grasp of pupils at an early point in secondary school mathematics. The procedure is a natural one for calculator or computer use. (Since this is another of those topics which could be the focus of an entire article, the reader is directed to two interesting papers, [9] and [10].)

The final topic is that of **mathematical modelling.** This involves notions of translation, but even more important, the development of equations if you will, which can be used to explain known phenomena as well as predict. The availability of calculators and computers will enable a student to work with and modify models; such activity has been hampered in the past because of the excessive calculations usually encountered. The NCTM 1979 Yearbook, *Applications in School Mathematics*, treats this topic in some depth (see my chapter).

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MYTHS ABOUT CALCULATORS IN THE SCHOOLS

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Every now and then a glimpse is caught by people outside the field, of how much more there is to mathematics than school arithmetic — it is art and science and it is creative, as well as logical and practical. And so it is also true that there is much more to arithmetic than computation. Since most people's math education was nearly all computation, most people are unaware of this broader definition. and the general public continues to demand that children spend most of their time learning the same complicated computational routines that they themselves were forced to learn during their elementary school years. In fact, this is now a waste of expensive teaching resources. From now on, repetitive drill and practice with computation techniques will be as worthless as they are demeaning. It should be clear that all that time on computation denies teachers and children a chance to participate in other areas of mathematics which are less mechanistic, more creative and far more practical.

In 1972, electronics companies introduced the four-function hand-held calculator at retail prices around S200. By 1976, the price of a calculator, now a more durable model, had dropped to about S10. It became an invaluable tool in business and at home. Its portability and responsiveness, as well as the ease with which it can execute computations, made its official or unofficial appearance in school inevitable.

Reaction to school use of calculators was emotional and divided. People favored total prohibition; most who could countenance it at all favored controlled and restricted use from fourth grade on, with little if any use in kindergarten through third grade. Very few people advocated unrestricted use of the machines. All in all, the calculator is having a hard time making its way into the elementary classroom.

The reasoning of the prohibitionists seems to us to be confused, short-sighted, or uninformed. Our guess is that, not having used calculators themselves to learn arithmetic, people feel anxious about allowing their children to use them because it seems to invalidate their own hard work in learning arithmetic. It is our intention in this paper to dispel anxieties. Perpetuated and unsupported fears become myths. The following are some of the most common myths about using calculators with children.

1. CHILDREN WILL BE HANDICAPPED BY THEIR DEPENDENCE ON CALCULATORS.

a. Should we give children crutches when they don't need them?

The calculator has been called a crutch primarily because it enables people to do computations that they are perfectly capable of doing or learning how to do without a calculator. We, however, have no objection to this use of calculators because valuable time will be saved. This will help shift major teaching priorities away from mechanistic, repetitive calculation. Today, most children leave elementary school with at least one of two severe mathematical disabilities caused by the current emphasis on computational facility. Calculators can be a legitimate remedy for either of these.

The most common handicap is the inability to solve simple real world problems, such as finding averages or selecting necessary quantities of paint. (1) The reasons for this handicap are not very complicated. Children spend 75% of the first seven years of elementary mathematics instruction learning and relearning the complicated, step-by-step procedures (called algorithms) necessary to do an addition, subtraction, multiplication, or division computation quickly and accurately. Then they spend the tiny remainder of their math time trying to learn a myriad of other things, including measurement, decimals, estimation, geometry, logic, statistics, real world applications, mathematical games, other-base numeration systems, and set theory. The traditional high priority of computation results in a population which is able to do a computation when specifically told which operations to perform on which numbers. Yet these people lack other mathematical competencies such as logical reasoning and the ability to apply that reasoning to the solving of problems. (2) In today's perspective, this deficiency is

(1) Carpenter, T. and T.G. Coburn and R.E. Reys and J.W. Wilson. Results from the First National Assessment of Educational Progress. (NCTM, 1978).

(2) Carpenter, T. and T.G. Coburn and R.E. Reys and J.W. Wilson. Results from the First National Assessment of Educational Progress. (NCTM, 1978).

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a serious handicap. A ten dollar machine can perform most ordinary computation, but society needs people who are able to clearly define a problem and a method of solving it.

The second handicap is the inability to do an arithmetic computation even when the numbers and operations are specified. For example, some children cannot successfully compute 346 x 28, 1869 ÷ 72, 4273 − 986 or 112.32 ÷ 89. These children are doubly disabled if, as is often the case, they also suffer from the first handicap. Calculators offer these children a powerful remedial aid or crutch. At least, if they are taught how to manipulate the calculator they will be able to compute.

For children, a calculator becomes a vehicle for communicating about numbers. Here, two 2nd graders from Denton, Texas, share a number discovery. (The boy on the left speaks little English; he's from Mexico and has been in the U.S. only a few weeks.)

b. What if a child needs to do a computation and doesn't have a calculator?

Consider what happens when you need to do an exact computation and you have no pencil and paper. Either you borrow them, or you save the computation until you can get your own pencil and paper. Calculators are now so inexpensive and ubiquitous that this analogy with pencil and paper is reasonable.

Certainly all businesses which need to do any calculations have calculators available. Even street vendors use them. It is just not efficient to do otherwise. On the other hand, if exactness is not required, then an estimate is needed. For this reason, we believe that the teaching of estimation in all the grades needs to be given a high priority.

c. What will a child do when the batteries die?

New types of calculators use far less energy. As a result, calculators are available that have a lifetime of 10,000 hours. At five hours per day of continuous use, seven days a week, that amounts to more than five years' time. Moreover, there are now slim, portable calculators with alarm clocks and stopwatches attached that are never turned off. The batteries are simply replaced on a regular schedule about once a year. This we feel is little maintenance for such a useful tool.

2. CALCULATORS SHOULDN'T BE USED BEFORE FOURTH GRADE.

a. Young children's minds could atrophy if they use calculators instead of thinking.

First, using a calculator does not diminish thinking. In order to enter data into the calculator correctly, students must think carefully about the related problem. This has a higher cognitive demand than does manipulating paper and pencil algorithms by rote.

Second, this statement assumes that the brain is a muscle that needs to be continuously exercised. Consequently, it is thought that doing complicated algorithms is good for the mind, just as lifting weights develops the biceps or running exercises the heart. However, the brain is not a muscle — it thinks whether we are conscious of thinking or not (for example, when we are dreaming). We realize that practicing computation can result in faster and more accurate computing; but there is virtually no enhancement of other mathematical abilities as a direct result of computational proficiency.

Further, it is commonly believed that whatever is hard to learn must be good and valuable. and what is easy to learn probably isn't valuable. We do not believe this. It is true that people value those things in which they have invested a great deal of time and energy. But if the same things can be accomplished more effciently, those long hours of work could be used on something else. Now that we have calculators to do the routine aspects of mathematics, it will be more important than ever to invest time and energy teaching creative thinking.

b. There are no good learning activities with calculators for young children.

This statement is usually based on the misconception that a child has to, or wants to, use a calculator as an adult does — as a fast and accurate computation machine. If you hand a calculator to an adult who has never used one before, after a very few minutes of routine manipulation that adult is bored. The feedback the adult receives is nothing new. This is not the case with children. For a child, a calculator is a responsive toy that almost says, "Play with me". Because of this our development group, EMC^2, has devoted four years to researching ways to use calculators with children of all ages. In kindergarten and first grade, before many children have mastered the fine motor skills necessary to write numbers with ease, a calculator makes it easy for them to communicate with and about numbers. This is important. Numbers become things with which they can do enjoyable things. The calculator is a writing instrument for them and can help make numbers more concrete and less abstract. At this early age, children enjoy counting with a calculator by 1's, 2's, 3's . . . . Counting, then, is a succession of "one mores", "two mores", "three mores", and so on. Thus, beginning even in kindergarten, counting can be taught as a foundation for addition and multiplication, rather than as a rote chant. In grades two and three, the calculator can help teach the similarity between one-digit and multi-digit multiplication. The paper and pencil ways of doing 15 x 8 and 15 x 24 are different, whereas the calculator methods are the same. In these grades, the connections between subtraction and division can be elucidated with a calculator. The relationship between an operation and its inverse can also be effectively demonstrated. With practice, problems involving large numbers may be tackled. Up to now this has been unrealistic. Finally, time can be devoted to solving problems — finding diverse solutions to one problem type as well as diverse problems that are solved by one solution type. In other words, various problem solving techniques and problem types can be examined if less time needs to be spent on pure computation.

Some educators argue that children have to understand the concepts of addition, subtraction, multiplication or division before they can be allowed to use a calculator. Here again is the misunderstanding of what it means to "use a calculator". We have noted several ways in which the calculator itself can help teach understanding of concepts. The calculator is not just used to apply concepts, it is used to understand the concepts. Understanding is especially important now that it is no longer necessary to focus solely on carrying out the operations.

Calculators might even allow teachers to look carefully at their children's private use of arithmetic and at the informal, intuitive strategies that children have invented or found outside of school. Recent in-depth interviews with children indicate that they make use of a rich repertoire of mathematical techniques to solve practical out-of-school math problems. (3) For example, counting on fingers to solve addition problems is one such technique which is, unfortunately, discouraged by most teachers. Few, if any, of these techniques were learned formally in school. These informal partial understandings need to be given validity by teachers and parents, if arithmetic is to be really useful to children. While children do use their incomplete understanding to do school math, it is rare that children make out-of-school use of the computation techniques they spend 75% of their in-school math time mastering. If calculators were to be adopted by teachers and parents as necessary learning devices, children would have a tool they could use easily, both in and out of school. It would connect in-school math with their out-of-school math. This could increase the relevance and decrease the drudgery of mathematics for children.

3. WITH CALCULATORS AVAILABLE, DECIMALS WILL BE EASY TO TEACH IN THE PRIMARY GRADES.

A calculator can display to any curious child numbers like 0.333 and 1.5. Thus, children will probably ask teachers questions about such numbers and perhaps be more motivated to learn about them.

Also because operations with decimals resemble operations with whole numbers, they probably will be easier to teach them operations with fractions.

However, the part-to-whole relationship is easier to visualize with fractional notation than with decimal notation. 1/3 represents one part of a whole which has been divided into three equal parts. The decimal equivalent, 0.333, is not easily interpreted as the same thing.

Further, increased computational facility will not necessarily enhance understanding of decimals. Ordering of decimals, for example, takes a good deal more understanding of place value than most primary-age children presently have. (Which is larger: 0.9 or 0.888?) Even with calculators, teaching decimals in the primary grades will require new materials and careful thought.

4. WITH CALCULATORS AROUND, THERE IS NO MORE REASON TO TEACH FRACTIONS.

Simple fractions, which become a part of our everyday vocabulary very early, are used quite often in our society and probably will continue to be used. "Half of the pie", "a quarter of an hour", or "a third of the money" are expressions which are not likely to disapoear with increased use of decimals. Hence, even in the elementary grades, we will need to maintain the skills of recognizing writing and ordering simple fractions.

In algebra, when one is solving equations, computing with fractions is as common as computing with integers. Operations like addition subtraction, multiplication and division with fractions could be performed more easily and more accurately on a calculator. However, students will still need to know when to operate on fractions, which operations are to be used, and how to manipulate the calculator to perform the operations.

It is true that most calculators display non-integers (like 2-3/4 and 1/6) as decimals. However, there are calculators that display and compute with fractions in either decimal or numerator-and-denominator form. If fractional notation is really important to the general public, all calculators could be manufactured with this capacity.

5. CHECKING PROBLEMS YOU HAVE DONE BY HAND AND DOING THE COMPONENT PARTS OF A LONG PAPER AND PENCIL ALGORITHM ARE VALUABLE USES OF THE CALCULATOR.

Using calculators to check hand computations teaches the child little if anything about mathematics. It does over emphasize the importance of the one right answer. But it reveals no additional information about the operation, especially since the calculator does not demonstrate to the user the component parts of its internal algorithm and the role that each plays. This repetitive process will only let students know if their computation is accurate. If the intent is to increase accuracy, then problems should be done on the calculator at the start and hand computation should be eliminated altogether.

Similarly, using the calculator to perform the component steps of a paper and pencil algorithm generally has little instructional value and is certainly not efficient for computation. It is somewhat like using parts of a good watch to make a sundial. Those who encourage children to use the calculator in this way with the division algorithm argue that understanding of the division process will be enhanced. However, simply ising the calculator for the intermittent multiplications and subtractions which occur in the long division algorithm will not necessarily illustrate the process of partitioning a set of objects, nor will it reveal the place value system of numeration that helps explain why the long division algorithm works.

6. THE STANDARD COMPUTATIONAL ALGORITHMS ARE BASIC AND IMPORTANT AND SHOULD ALWAYS BE TAUGHT.

(3) Ginsburg, Herbert; "The Psychology of Arithmetic Thinking". The Journal of Children's Mathematical Behavior, Spring 1977, pp. 1-89.

For hundreds of years around the time of Christ, people used the Roman numeral system of naming numbers. Anybody who needed to multiply whole numbers greater than one hundred had to translate the problem from the Roman notation onto an abacus-like device, do the computation on the abacus, and carefully translate the result back to the Roman system. This method was ingenious but time-consuming to use and difficult to learn.

When the Arabic numeral system (our present base-10 place-value system) became known, the computational algorithms became simpler to learn and use. Indeed, as we mentioned previously, most children can use our computational algoritms reasonably well by the time they leave eighth grade. But the primary purpose of these algorithms has always been to get fast and accurate answers to quantitative questions. The calculator is a better algorithm in this sense than are any of the paper and pencil algorithms we now teach, just as the place value numeral system was better than the Roman system. The calculator is faster and more accurate and it requires less effort to learn. Some of the algorithms like the long division or long multiplication algorithms are more difficult for young children to learn than the actual concepts of multiplication or division. Consequently, computational facility comes only after a number of years of learning and relearning. But, most important, there is no indication that facility with an algorithm enhances understanding of the related concept. Listen to a child or to yourself as you go through a multiplication procedure to do 365 x 748: "Eight fives are forty, put down a zero, carry a four; eight sixes are forty-eight and the four makes fifty-two, put down the two, carry the five," etc., etc., etc. This is simply a recitation of a previously memorized algorithm to do the multiplication. In fact, the purpose of memorizing it so carefully is so we do not have to reason our way through the algorithm each time we use it. That's why algorithms are useful. We drill them into ourselves (and our children) precisely because we want to be able to complete the computation quickly and accurately with as little conscious effort as possible. However, most adults could not explain why and how the procedures work. Thus, if they forget one step in the procedure, they may have sufficient understanding to reconstruct for themselves the procedure needed.

We must conclude then that what is basic and important is understanding of the problem and possible ways of solving it, rather than understanding of the specific algorithm. If computation is necessary for determining a solution, then a calculator can and should be used.

Conclusion

We have stated some of the common fears that prevent school use of calculators and we have mentioned a number of positive ways calculators can be used in the primary grades to teach new concepts rather than simply to reinforce those already learned.

No longer does arithmetic have to be synonymous with paper and pencil computation. In fact, all hand computation probably will be replaced by calculator computation soon – if not in one year, then in fifteen years. The implications of these facts are that teachers need to accept the reality of the calculator and to change their emphasis and focus. Since it is known that computational ability is not closely related to problem-solving ability, (4) teachers have little to fear from using the calculator for all computation, and good reason to look forward to devoting more time to developing other mathematics skills and abilities.

(4) Begle, Edward; "Some Lessons Learned by SMSG". The Mathematics Teacher, March 1973, pp. 207-214.

Section III

CALCULATORS

and

PROBLEM-SOLVING

Using Calculators for Problem Solving

by JEAN B. ROGERS

Jean B. Rogers is a member of the Executive Council of the Oregon Council for Computer Education and is active in computer and calculator circles in Oregon. – Editor

This unit is designed to present the user with information on how calculators can be used in mathematical problem solving methods. The two techniques involved are TRIAL AND CORRECTION and MAKE A TABLE, both easily and commonly used.

After completing this unit, the user:

a) will be able to apply two specified problem solving methods to appropriate problems.

b) will be aware that a calculator is useful when these methods are applied to mathematical problems.

The unit presupposes an ability to understand what a square root is, and the ability to calculate simple interest. A basic four-function calculator is the only tool required.

It is important to emphasize that the methods presented in this material are general problem solving methods, and not restricted to the specific types of examples given.

CALCULATORS AND PROBLEM SOLVING

A calculator is a tool that can increase the problem solving power of the user. There are many techniques that are helpful when we approach a problem, be it mathematical or non-mathematical, and knowing these general methods increases the ease with which we solve the problem. A calculator can make some of these techniques even better when we are working on a problem involving numbers.

TRIAL AND CORRECTION

One of the most commonly used problem solving techniques is called "trial and error", but to take a more positive approach, let's call it TRIAL AND CORRECTION. A calculator makes it so much simpler to do the "trials" that this becomes a very efficient way to work out a problem.

For example, let's look at finding the square root of a number. We know that we can find a square root by using an algorithm (a step-by-step procedure that will determine the result), if we can remember it. For square root it looks like this:

$$\begin{array}{r|l} & 15 \\ & \sqrt{225} \\ \underline{1} & \underline{1} \\ & 125 \\ \underline{25} & \underline{125} \end{array}$$

However, with a calculator to increase our power, we could take a different approach: TRIAL AND CORRECTION. To do this we are going to use a machine with just the four basic functions on it. If yours has a square root key, just ignore it for now,

A way to find the square root of 3136: ($\sqrt{3136}$)

Choose a number, multiply it by itself, and if the result is 3136, that number is $\sqrt{3136}$.

TRIAL NUMBER	TRIAL TIMES TRIAL	
50	2500	Too little
60	3600	Too big
55	3025	Too little, but close
** 56	3136	

After each trial, I used the information yielded by my multiplication result, either too little or too big, to correct my old trial so that the next trial is a better one. Thus we have the TRIAL AND CORRECTION method.

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Use the TRIAL AND CORRECTION method to find: the square root of 729
the square root of 42.25

$\sqrt{729}$		$\sqrt{42.25}$	
TRIAL NUMBER	TRIAL TIMES TRIAL	TRIAL NUMBER	TRIAL TIMES TRIAL

MAKE A TABLE

Another problem solving technique is to MAKE A TABLE, and this is also a place where a calculator can be helpful. Having a table is very beneficial when values are changing while a process is being carried out, or when there are many values that need to be available for comparison. Calculating interest on a loan or a savings account is a good time to use this method.

Suppose I make a credit purchase for \$70 and pay it off at \$10 a month. Since I must pay 1½% interest on the remaining balance each month, only the remainder of my \$10 payment is credited to my account.

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8
Balance	\$70.00	\$61.05	\$51.97	\$42.75	\$33.39	\$23.89	\$14.25	\$4.46
Interest	1.05	.92	.78	.64	.50	.36	.21	.07
Credited to account	8.95	9.08	9.22	9.36	9.50	9.64	9.79	****

**** Last payment is balance plus interest.

From this table we see that it will take 8 payments to pay the \$70 debt. By summing the entries in the Interest row of the table, we find the the total interest cost will be \$4.53.

Make a similar table for payments of \$20 per month.

	Month 1	Month 2	Month 3	Month 4
Balance	\$70.00			
Interest				
Credited to account				

From your table you should find that 4 payments are required with an interest cost of \$2.48.

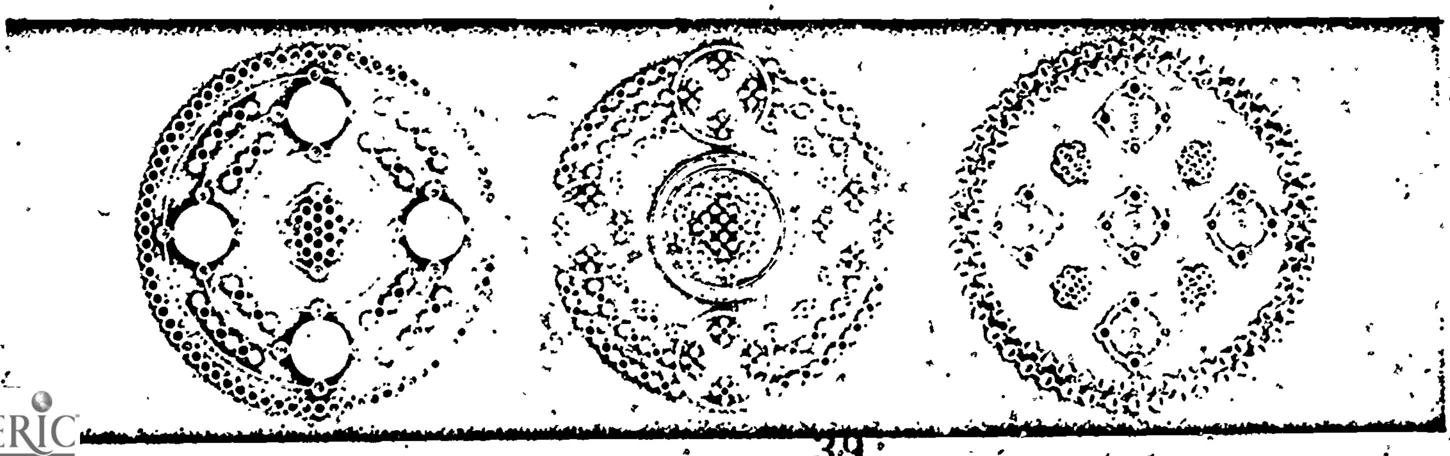
Make two more tables, finding similar data for a \$70 debt if the interest rate is only 1% while the payments are again \$10 per month and \$20 per month.

Now make a summary table that shows the interest cost and number of payments for the four different circumstances we worked out above. This final table will give you an easy way to compare the data on the four plans.

	Total Interest Cost	Number of Payments
\$10 payments at 1½%		
\$20 payments at 1½%		
\$10 payments at 1 %		
\$20 payments at 1 %		

CONCLUSIONS

We have seen how a calculator can be used in two problem solving techniques, TRIAL AND CORRECTION and MAKE A TABLE. Remember that the examples we looked at are only a few out of the many different types of problems to which these methods can be applied, and that there are many occasions when these techniques can help people increase their problem solving power.



Mini-Calculators And Problem Solving

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During the past five years, one of the most widely discussed topics at any meeting where math educators gather has been the impact of the minicalculator on the classroom environment. "Do children become dependent on calculators?" "How can calculators be used constructively?" "Are calculators durable enough for classroom use?"

University educators were not allowed their leisure at coming to grips with the problem. Manufacturers were producing hand calculators within the price range of many families in a short two or three years after the question came to national attention. Students brought them to class and teachers wanted direction immediately. Articles, pro and con, appeared often. State, Regional, and National meetings were filled with sessions, workshops, debates, and research concerning the use of these tiny marvels. The Board of Directors of the National Council of Teachers of Mathematics adopted a position which read in part:

> Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom the mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics. (NCTM, *Newsletter*, December, 1974)

The proclamation which appeared less than five years after the issue became prominent, gave a great deal of support to the protagonists of this device.

Today, most mathematics educators have accepted this position (some, less eagerly than others), either because of continuing research supporting the use of hand-held calculators, or because rejection would be tantamount to burying ones head in the sand. Several studies (Rudnick and Krulik, 1976; Schnur and Lang, 1976) have demonstrated that students who use mini-calculators do no worse, and in some cases improve significantly, in overall mathematical achievement than youngsters who do not use calculators. School districts in New York, California, Indiana, and other states across the Nation are putting calculators in a number of experimental classes. Preliminary results seem to indicate that students take an interest in using minicalculators and that pencil and paper computational skill test scores are as high for students who regularly use calculators as those without access to these electronic instruments.

Such widespread acceptance is by no means a fact of life among parents. Rudnick and Krulik (1976) indicate that the majority of parents

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whose children participated in their study had sincere reservations concerning the use of hand-held calculators in the classroom. They reported that parents were overwhelmingly certain that children would become dependent on calculators with continued use.

In the last five years, authors and speakers have continued to suggest uses of the calculator in "imaginative and enjoyable" ways. Many of these uses seem to focus on the motivational value of the calculator, stressing games or computations necessitating their use. Somewhat less is said about their functional use (as a tool). A consensus among these vocal leaders of the movement is that the word problems in the texts become almost trivial when calculators are used in place of pencil and paper algorithms and echo the need for more meaningful problem solving experiences. Indeed, widespread use of the hand-held calculator may shrink the pencil and paper algorithm curriculum, but it will certainly expand the curriculum in the direction of real world problem solving.

Consider the model for human problem solving based on an informational processing theory which views problem solving as sequencial process. The solver defines the problem, decides on a plan of attack, carries out this plan and chooses the optimal solution. There is evidence (Shapiro, 1973; Shields, 1976), to suggest that children d[illegible]e problems using a discernible pattern, and that solutions to comprehensive problems become more realistic and complete as age and problem solving experience increases. Hence, it seems reasonable to begin comprehensive problem solving practice in the early grades and continue such efforts throughout the school experience. It is within the context of such practice that hand-held calculator may have its most valued impact. Students using calculators as their adult problem solving counterparts use them, as a tool.

Consider the problem solving process in greater detail. During the first step, the solver recognizes the question being asked and translates the problem into terms that make sense to him. Step two, development of a plan of attack, often involves the use of one or more generalized problem solving strategies. These "rules of thumb" include working backwards, using contradiction, identifying subgoals or looking at a related problem.

After the solver has some plan, he proceeds with this plan to reach one or more possible solutions. This third ~~step in the solving~~ process involves the use of a number of heuristics which are taught in the elementary schools. For example, the solver may make a list or table, guess and test various hypotheses, use a graph or draw figures and set up equations. It is this step of the solving process in which the mini- calculator will aid students in reaching more accurate and realistic solutions. In addition, during the fourth step in the process, selecting the optimal solution, the calculator can be used effectively to check the solution against given information.

For example, in one sixth grade class, a discussion concerning the safety of an elevator in a nearby department store prompted the teacher to challenge the class to determine the number of persons who could safely use the elevator, which had a posted weight limit of 1600 pounds. After some additional discussion, it was determined that the solution depended on the "average" user of the elevator. Two days later, groups of students posted themselves outside the elevator to ask users how much they weighed. After several days and many weights, the average weight was determined with the aid of calculators. The sample of over one hundred respondents was handled easily using calculators. An added bonus was that the youngsters were able to average after each group had completed its questioning, and found that after the first fifty weights, the average weight fluxuated very little with additional data. (This is an excellent example of the Law of Large Numbers, that is, $\frac{S_n}{n}$ tends to stabilize as n becomes large, which an alert teacher could exploit to teach some basic probability). After it was determined that ten and a half "average" people could use the elevator, the discussion began to focus on the accuracy of this solution. Some students noted that some individuals questioned did not respond; other students wondered about people who lied about their weight. The consensus was that some safety margin should be allowed. Finally, the number of people, (men, women, and children), was limited to ten, and further discussion was postponed for a few days.

When the elevator problem was again raised, a young man suggested that they do some investigation concerning weight variations for the population using the elevator. Certainly ten businessmen could exceed the 1600 pound weight limitation, whereas, this limit probably would never be reached by ten sixth graders. For this part of the problem the students used a table of heights and weights, rather than sampling individuals at the elevator. Using the range of weights indicated for the adult male, female, and eleven year old boys and girls, the students constructed confidence intervals for the safe number of elevator users. Their conclusions are shown in Figure 1.

Boys, 11 years old	[17,25]
Girls, 11 years old	[17,24]
Adult females	[10,13]
Adult males	[7,9]

FIGURE 1—Confidence intervals for safe number of elevator users.

The students could not use this additional information, except to agree that a "surefire" safe limit could be set at seven—the minimum allowable riders for the four populations studied.

This final solution to the problem gave rise to some additional questions: "Since we have a safety margin in our limit, did the elevator manufacturer also include a margin of safety in the 1600 pound limit?" Maybe the elevator won't hold (accommodate) seven very large men." Discussion continued until the bell signaled the end of class. Interest in the problem diminished and the challenge did not come up for general discussion again. Perhaps the students recognized that they had achieved only a partial solution, but one they could live with.

This example provides several instances where the calculator insured accuracy and, more importantly, prevented the solving process from becoming "bogged down" in laborious algorithms. Often, young children will spend a great deal of time on pencil and paper algorithms, and forget the purpose of their calculations.

If our intent is to provide experience in problem solving so that students gain insight into the process, then we should minimize the number of instances where children can become sidetracked. In our example, the numerical solution is not nearly as important as learning that any law which guarantees the safety of everyone in a particular instance can become overly restrictive. This class was beginning to recognize this problem while being ensconced in the solving process. This type of insight and involvement is necessary to provide for transfer to other similar problem solving situations.

The kind and number of comprehensive problems which can be constructively undertaken by children is limited only by your imagination. Some popular challenges which children enjoy are design problems. Design a playground, park, or bike trail. Children may elect to improve the lunch room service or the crosswalk in front of the school. Any problem which your students attempt to solve should embody the following principles:

1. It must be of interest to youngsters so that they are willing to undertake the time consuming problem solving task..
2. It must be a situation or question for which the students do not already possess a solution, or a direct method of obtaining the solution (e.g. solve a readily available equation).
3. It must be solvable using previous learning.
4. If possible, the solution should become reality.

The first three conditions insure that youngsters work at the highest cognitive level creating something new in their experience. The fourth condition is necessary so that children will accept future challenges. One fourth grade class tried to solve the continuing logistics problem in the school cafeteria and worked on the various aspects of the problem for about six weeks. The culmination of this work was a collection of proposals which they expected would "improve the service in the cafeteria". This list was taken to the principal who took a great deal of time to evaluate, suggest modifications, and present the proposals to the required administrative personnel. As a result of their work, several changes were implemented, and the students were anxious to attack other problems they found around the school.

The Unified Science and Mathematics for Elementary Schools (USMES) project has developed and tested several interdisciplinary units centered on long-range investigations of real and practical problems taken from the local/community environment. Some of the units involve challenges in advertising, bicycle transportation, designing for human proportions, consumer research and soft drink design. These units, each embodied in a teacher resource book, have been classroom tested by teachers from all over the country during the past five years and have proved popular with students and teachers alike. Any of these problems or the comprehensive problems defined by you and your students will provide hundred of computational task over the many weeks it takes to reach a satisfactory solution.

Consider the following comprehensive problem which children enjoy: Invent a new soft drink that would be popular and produced at a low cost (USMES, 1973). Typically, an intermediate grade or middle school class will spend two or three periods per week, for several weeks, reaching some solution to this question. During that time interval students will conduct surveys on taste, carbonation, and price. They will test drinks for individual preferences, and devise a rating system for soft drinks. They must analyze taste results for intermediate and final experimental drinks. In one particular class, the students used this data to find means, variations, percentages and cost analysis. In addition, two youngsters tried to find an equation which predicted the cost-preference relationship. Although their efforts were fruitless, their teacher reported that they tried such a task because of their belief in the power of their calculator. Therein lies one of the most striking arguments for using hand-held calculators in the context of comprehensive problem solving: the solver learns the limitation of the computing device and begins to understand the need to organize and direct the solving process. Indeed, one of the most crucial ingredients in effective problem solving is the organization of, and inferences made from the given information.

How does the teacher plan her instructional sequence so that her class is in a position to take advantage of the symbiosis of problem solving and the hand calculator? Before any interesting problem solving can occur, the student has to be conversant with the calculator and the "fun time" atmosphere prevalent when calculators are first introduced must diminish. The student needs a little instruction and some time to practice punching the correct buttons. A successful way to provide adequate preparation is through games and large number calculations.[1] Concurrent with this practice, the teacher should supply some short puzzle problems from which their pupils can learn the organization of data and generalized problem solving strategies.[2] This simultaneous practice may continue as long as your students enjoy and profit from the experience, but it should not keep you from getting into comprehensive problem solving within a few weeks after the calculators are introduced into your classroom.

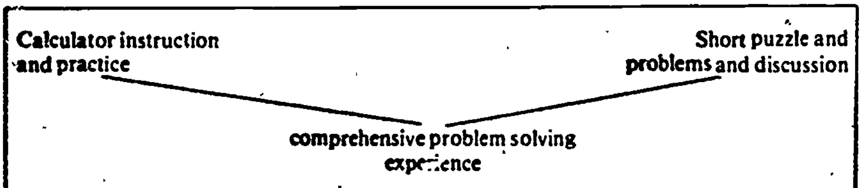

FIGURE 2—Sequence for introducing problem solving using calculators.

Once a real world challenge is introduced, allow your students to find their own direction without letting them stray from their intended goal. Most teachers find that class discussion prior to data gathering, computations, experimentations, etc., prevents the thrust of such work from becoming divergent. Teachers should not discourage enthusiasm in a direction which is foreseen as an unproductive search. Failure in one direction may be as instructive as continued teacher engineered success. Periodic work on the challenge should continue until interest begins to wane in the majority of your youngsters.

Summary

Our focus in this article has been to show that mini-calculators and problem solving, two of the most topical issues in mathematics education today, complement each other perfectly. The natural combination of real world problem solving experience and the hand-held calculator provides an answer to the recurring teacher question, "Now that we know how to push the buttons, what do we do?".

Comprehensive problem solving supplies many realistic computational situations where youngsters can use their hand-held calculators. Further-

1. Many examples of these kinds of activities can be found in the *Arithmetic Teacher*, November, 1976; Vol 23, No. 7.
2. For example, see the *Basic Thinking Skill Series* by Anita Harnadek, Midwest Publications Co., Troy, Michigan: 1977).

more, in the context of this problem solving experience, children begin to appreciate the strengths and limitations of the mini-calculator.

On the other hand, the calculator aids pupils in reaching accurate solutions to computational questions quickly. This speed and accuracy helps them get through the forest of problem solving without getting sidetracked while they are working around a single tree. So wait no longer, start using those miniature marvels in meaningful ways through investigations of long range comprehensive problems.

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Calculators: Their Use In The Classroom

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The use of pocket calculators within a classroom setting has received a great deal of attention in recent months. Briefly, the proponents of these devices argue that calculators can: 1. reduce the time spent on tedious and routine computation; 2. allow its users more time to focus on problem solving techniques; and 3. provide increased motivation toward the study of mathematics.

Opponents of the use of calculators in the classroom fear that these devices may become a crutch and thus students will become weak in their ability to perform basic arithmetic computation. This philosophy proposes that students may become so dependent on their machines that once removed, a student may not be able to independently execute basic number skills.

The solution to this controversy rests with the investigation and promotion of various experimental programs involving calculators. Any weaknesses or imperfections in existing programs must be ironed out so that those with justified criticism can accept the use of calculators as a worthwhile learning device in the classroom.

PURPOSE

In November, 1974 a proposal was approved by the Bethlehem Central School District, thus inaugurating a program utilizing minicalculators in mathematics class at the eighth grade level.

In accordance with the National Council of Teachers of Mathematics policy statement on the utilization of calculators in the classroom, our program was intended to reinforce learning and to motivate the learner in the study of mathematics. The function of the calculators were: 1) to provide continued and increased motivation toward the subject matter; 2) to furnish faster and more efficient ways of solving problems; 3) to allow problems of greater intricacy to be attempted; and 4) to contribute to further applications and exploration of related topics.

It was our belief that the use of a calculator was both a faster and more efficient means of achieving solutions to mathematical problems. However, the calculator was not meant to be a substitute for skills which otherwise could be performed manually. In order to operate such a device, the student must possess a basic understanding of the mathematical concepts of addition, subtraction, multiplication, and division. Therefore, the use of a calculator was not an end in itself, but a means to an end. The emphasis in the classroom was to improve the problem solving skill rather than their ability to compute.

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PREPARATION

Our program proposal called for the purchase of fifteen Bowmar-25 calculators and adaptors. At the time of purchase in November 1974, the cost of each calculator and adaptor was approximately thirty-five dollars. However, present market prices of calculators have shown a substantial reduction over the past two years. One calculator and adaptor similar to the ones we have employed in our program can be procured for under twenty dollars.

The physical requirements for calculators to be utilized in a classroom setting were minimal. A locking two drawer file cabinet was used to store all equipment. The top drawer of the cabinet was divided into fifteen numbered areas corresponding to the specific number on each adaptor. Similarly, the bottom drawer was subdivided and labeled according to the same number imprinted on each calculator.

In our program we opted for adaptors over the more conventional recharging devices which other calculator programs seemed to prefer. We felt that since we had fifteen calculators which were in operation during five different class periods in a school day, adaptors would be more advantageous. Recharging devices could conceivably lose their charge during the day through overuse or misuse. It was our belief that recharging devices would have been more valuable if the calculators were used by a small group of students. In our case, adaptors provided a continuous and longlasting performance to a large number of students.

Our final barrier in preparing the classroom for implementing calculator usage was solved with the assistance of our school electrician. The classroom had only two electrical outlets but our electrician designed three twenty foot extension cords with outlets every four feet. Thus, each extension cord was planted between a set of two desks and each cord provided five extra outlets in the room from which a source of current could be drawn.

CLASSROOM OPERATION

In utilizing calculators in a classroom environment certain regulations were necessary to insure the systematic operation of the program. In our program, rules and regulations regarding the use of the calculators were specifically outlined and strictly enforced. Students were fully aware of their responsibility in taking care of their assigned calculator. This included careful handling of the calculator and adaptor at all times, sharing the calculator with a respective partner, and other applicable instructions. We found that students rarely took advantage of the situation since misuse of a calculator resulted in a loss of privileges for those individuals.

In each mathematics class where calculators were employed, one responsible student was selected to assist the teacher in the distribution and collection of machines. The dual function of the student monitor was to carefully distribute the appropriate calculator to individual classmates as their number was called by the teacher. Upon receiving each calculator at the end of a class period, the student monitor returned each calculator to its proper resting place and checked to see that all machines were properly turned off. The teacher was responsible for the distribution and collection of each adaptor.

The student monitor was an energetic contributor to the efficient functioning and overall success of the calculator program. By having an able assistant, the teacher was in a better position to control the completion of activities for that classroom period.

Utilization of calculators in mathematics class began with instruction on how to operate the calculator. Initially, ten minutes of free time was set aside for students to explore the capabilities of the calculator.

As the popularity of calculators has continued, many companies have produced various models. In our program we had to overcome the fact that a large percentage of students had access to calculators at home. More often than not, differences did exist between the calculators students had used previously and the Bowmar-25 calculators. However, a model calculator made of construction paper (1 ½ feet by 3 feet) was employed to explain all capabilities and limitations of the Bowmar-25.

Although many students claimed to be proficient in the use of a calculator, we found that at least two full class periods devoted strictly to problems in addition, subtraction, multiplication, and division were necessary to insure mastery of fundamentals. In particular, division problems proved to be most difficult for students to solve. For example,

$.3\overline{)432}$, $\frac{432}{.3}$, or $432 \div .3$ all represent identical division problems but on a calculator many students often confused the order of imput. As a result, a great deal of practice in addition, subtraction, multiplication, and especially division was provided before proceeding to specific problem solving situations.

USES IN MATHEMATICS CLASS

As we approach the end of our fourth year of using calculators in mathematics classes, there does exist one method of operation superior to all others. We have experimented with the use of calculators on a daily basis for a short period of time, as well as the utilization of these devices on a once a week basis. It seems that no matter how familiar students become with the use of calculators, new material could not be presented in a class period where the calculators were in operation. The level of maturity of eighth grade students combined with the distracting powers of the calculator presented too great a hindrance to allow normal learning activities to take place. However, we did conclude that in order to profit most from the use of calculators in a mathematics program, usage of the devices should take place once a week.

Usually at the end of each week, following a fifteen minute quiz reviewing the material for that week, the calculators were used to solve many related problems. In this case, any explanation of a problem by the mathematics teacher could generally be bypassed since in the minds of most students the material was very fresh. This technique prevented students from becoming too dependent on the calculators, but at the same time provided increased motivational support for the required curriculum.

The eighth grade curriculum at the Bethlehem Central Middle School is composed of many diversified topics representing various branches of mathematics. Units of study covered during the school year include probability, statistics, proportion, percent, coordinate geometry, solid geometry, and algebra. Within each unit there exists several exercises involving the use of calculators which serve to reinforce learning activities from previous class periods.

By way of example from the unit on solid geometry, the volume of a rectangular prism and the volume of a cylinder were studied during one week of classes. In class on Friday students were tested via a short quiz reviewing the material on volume covered during the previous four days. In the remaining twenty to twenty-five minutes of class, students were required to complete an exercise with the aid of calculators. The exercise involved calculating the volume of two rectangular prisms and four cylinders. Students were given the dimensions of each figure and formulas were written on the board for reference. The substance of that exercise is summarized below.

FIGURE	LENGTH OF BASE	WIDTH OF BASE	HEIGHT	*VOLUME = L x W x H
1) Rectangular Prism	19.85 ft	11.3 ft	7.7 ft	1727.15 cu ft
2) Rectangular Prism	42.3 cm	39.6 cm	15.5 cm	25,963.74 cu cm
	RADIUS	HEIGHT		VOLUME = $\pi R^2 h$
3) Cylinder**	7.4 in	16.7 in		2871.50 cu in
4) Cylinder	29.5 cm	52.8 cm		144,280.48 cu cm
5) Cylinder	.54 m	1.6 m		1.46 cu m
6) Cylinder	72 ft	125.8 ft		2,047,742.2 cu ft

*Students were instructed to round off answers to the nearest hundredth place.
** $\pi = 3.14$

The preceding exercise satisfies the objectives stated earlier in this article concerning the function of calculators in a classroom setting. The use of calculators to assist in problems on volume illustrates that more intricate and exact dimensions of geometric figures which ordinarily have not been part of the curriculum can be included as required classwork.

One of the goals of our program was to include exercises involving the use of calculators which reinforced recent classroom activities. In achieving this goal, the facilitation of student interest in mathematics has also been increased. The implementation of a faster and more efficient method of problem solving through the utilization of calculators substantiates still another advantage of our program.

SUMMARY

The use of calculators in the classroom adds a distinctive feature to any mathematics program. Based on four years of experience, it is the feeling of the author that calculators have a definite function in a classroom setting. At the eighth grade level calculators, if available, should be an integral part of the mathematics program but only as a supplemental aid to learning.

The following list of guidelines is meant to assist those educators in the process of initiating a program involving the use of calculators.

1) The purchase of all calculators should include a one year warranty to replace or repair any malfunctioning machine.
2) Distinct and permanent identification is necessary for all calculators and adaptors.
3) The authors highly recommend the use of electrical adaptors as opposed to any type of recharging device. Adaptors will provide uninterrupted and longer lasting service in the utilization of calculators.
4) A locking cabinet must be provided to enhance the easy distribution, collection, and protection of all calculators and adaptors.
5) Designated calculators should be assigned to students so that a particular machine is utilized by the same pair of students on a continuous basis.
6) Rules and regulations involving the use of calculators must be clearly stated and enforced so that students will exercise care in the operation of each calculator.
7) A trustworthy student should assist the teacher in the distribution and collection of calculators during a class period.
8) At least two full class periods of instruction should be provided to all students vis-a-vis methods of operating a calculator.
9) Although educators should be urged to explore all avenues of incorporating calculator usage into daily lessons, we highly recommend that the utilization of calculators not exceed one experience per week. The novelty of calculators in a classroom environment can easily be eroded by overuse that more importantly basic computational skills might eventually become weaker.

Enrichment problems dealing with such topics as probability, statistics, proportion, and geometric figures illustrate how easily calculators can become a valuable tool in the study of mathematics. As an instrument of great motivational prowess, the incorporation of calculators into a mathematics program is rapidly and deservedly gaining prominence as an essential resource.

Section IV

USING CALCULATORS:

Suggestions from Surveys, Research and Classroom Trials

A CASE FOR THE CALCULATOR

By Glen D. Vannatta and Lucreda A. Hutton

During the 1976–77 school year, 560 hand-held calculators were introduced into the mathematics instruction of thirty-eight intermediate-level classrooms in six Title I schools in the Indianapolis Public School System. Approximately 1000 students were involved and all teachers of fourth, fifth, and sixth grades in the six schools participated in the project. The project was continued during the 1977–78 school year, but during the second year teacher participation was entirely voluntary.

The purpose of this project was to investigate the possibility of improving problem-solving performance and increasing student interest in mathematics by incorporating calculators in the mathematics curriculum.

Classroom Materials and Methods

To enhance the use of calculators, a variety of written materials was provided to project teachers. A publication titled "How to Use Your Mini-Calculator" was prepared and distributed for orientation and familiarization of teachers and their students. Also, a guide correlating calculator use with the adopted mathematics textbooks was given to each teacher. The guide proceeded page by page through the textbooks and gave directions and advice on the many ways of using the calculators as an integral part of instruction. The primary uses of the calculators included reinforcement of computational skills, textbook problem solving, supplementary practice with large numbers, and extended out-of-text problem solving.

In addition to the guide, a correlated series entitled "Problem of the Week" was developed for calculator solution. These problems concerned applications that were relevant and of current interest to intermediate-grade pupils. For example, during the very cold winter period, one problem concerned earning money by shoveling snow. The problems were graded in difficulty so that fourth, fifth, and sixth graders could work on the same general topic but at different levels of complexity. Each "Problem of the Week" was put on transparencies so that teachers could easily present the problem to an entire class for discussion and calculator solution.

Teachers had considerable latitude in the manner of their use of the calculators. Pupils worked in pairs for most calculator periods. One pupil would work a problem while the other observed and made a written record of the work. Then an exchange was made. In some activities the pupils played a competitive game. For example, pupil A would enter a multiplication fact such as 7×9. Pupil B would have to say aloud the product and then push the = key to reveal the correct answer. A point was scored for pupil B if he or she was correct, or a point for pupil A, if not. Pupils alternated in giving problems until a goal score was reached. In this way skill in basic facts was developed or reinforced.

Teacher cooperation in the first year of the project was highly variable. Although there was no open refusal to participate, it was evident that a few teachers did not support the premise of using calculators in mathematics classrooms to improve problem-solving abilities. Others were very positive and creative in fulfilling their obligations. One teacher developed a series of lessons on nutrition that involved problems about calories, vitamins, size of servings, and so on. The integration of subject matter was excellent, and the calculators were used to arrive at conclusions that were interesting and meaningful.

Equipment

The calculators chosen for the project were basic four-function machines with memory, percent, square root, and sign-change keys. The keys were large and color coded. Wooden boxes capable of holding one classroom set of calculators in compartments were made by the industrial arts department. Each calculator was coded to indicate school, teacher, and student. Varying routines of checking out and checking in the calculators from the boxes were developed by the teachers. The identification numbers and pupil monitors proved very effective. The matter of security was less serious than anticipated. Perhaps we were lucky; but only two calculators were missing at the end of the second year, and these were lost in a school break-in, not as a result of careless classroom management. The calculator boxes were usually stored in a lockable cabinet or in a closet when not in use. Only one calculator was damaged due to the extent of malfunction by pupil abuse. About four percent of the calculators developed malfunctions due to defective manufacture and were replaced under warranty.

Another concern centered on the power source for the calculators. The calculators used were powered by four AA disposable batteries. Battery consumption was less than anticipated. An

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estimate of battery cost is between $1 and $2 per year per calculator for moderate or normal usage. As a backup, AC adapters were included in the purchase price of the calculators but were not distributed or used. The specter of a tangle of extension cords, the hazard of electrical outlets in antique classrooms, and security problems limited the usefulness of adapters in our judgment.

Project Results

After two years of operation of the Indianapolis Public Schools calculator project, we reached several conclusions, including the following:

1. Management, security, and power source are minor concerns that can be easily handled.
2. Most pupils *are interested* in and motivated by calculators.
3. Most pupils *can learn* to use calculators quickly, carefully, and accurately.
4. During the 1976–77 school year the sixth-grade students registered a significant increase in problem-solving performance.
5. During the 1977–78 school year calculator classes showed achievement well above normal expectations in both computation and problem applications on the California Achievement Test.
6. Use of calculators in intermediate-level classrooms does not seem to result in a decline in computation performance. Although during the 1976–77 school year the fourth-grade students registered a significantly smaller gain in computation performance than did the control group, the 1977–78 results show no indication of a decrease. At the end of the two year period, improved performance was noted for most classes.

The California Achievement Test was administered to the calculator classes in September 1977 and again in May 1978. For this period a grade equivalent score difference of 0.8 could be expected in combined computation and concepts/applications. The outcome observed was that 86 percent of the calculator classes scored mean grade equivalent gains greater than 0.8. Actually, in 1977–78, computation gains exceeded those for concepts/applications. No control group data were available during the second year of the program. The results were similar for each of the three grades, fourth, fifth, and sixth.

There are many factors that affected student performance. It is not our intention to claim that the impressive gains were solely the result of calculator use. Other factors were present, as in any typical school setting. During the first year the research design included a control sample. The Metropolitan Achievement Test was administered for evaluation of computation and problem solving performance. The second year did not involve a research design with a control sample and the California Achievement Test was administered for evaluation of computation and concepts/applications. It is a fact, however, that the calculators were available and were incorporated into the mathematics instruction of these classrooms and the students did show very good gains on the California Achievement Test subtests on computation and applications.

The teacher factor remains the most important aspect of effective instruction—with or without calculators. The teachers need and want curriculum support from the administration. In our opinion, a successful calculator program must include effective teaching materials correlated with the ongoing mathematics program as well as calculators.

If this study has value for elementary teachers, it is the positive indication that they can experiment with calculators in the classroom without fear of endangering computational skills, and that a thoughtful program of instruction integrating calculators with text material can be presented. The only danger, it seems to us, is to place calculators into a classroom and leave the outcome to chance.

Editor's note: As this issue goes to press, the authors report that the calculator project in the Indianapolis Public Schools continues alive and well. Three other Title I schools have been added in an expansion of the project. Although no formal evaluation has been undertaken, feedback from teachers and standardized testing gives evidence that the goals and approach are effective. Another calculator project for high-achieving pupils in grades four through six has adopted some of the same features and materials of this first project. The great importance of in-service training of teachers in instructional uses of calculators has also been demonstrated. □

Hand Calculators What's Happening in Schools Today?

By Robert E. Reys, Barbara J. Bestgen, James F. Rybolt, and J. Wendell Wyatt

What's happening with calculators in school's today? Are they being used? If so, by what students? How do teachers feel about using calculators in the mathematics program? Should calculators be used on standardized tests? Should use of calculators be integrated into basal mathematics textbooks? Accurate answers to such questions are essential in assessing the current status of calculator use in schools today and more importantly, preparing for calculator usage in the mathematics curriculum during the 1980s.

During the 1970s many voices have been heard debating the virtues and dangers of calculator usage. A few have cried out in fear that use of calculators will result in pupils who cannot remember basic facts or do traditional paper-and-pencil computation. Teachers, in particular, are concerned about how calculators will affect students' computational skills (Palmer). The "rot-the-mind theory" has not been supported by research (Suydam). Although long-term effects of sustained calculator usage are not yet known, there is ample evidence that frequent use of calculators in elementary schools has no detrimental effect on achievement in mathematics. Some have proclaimed that the rapid growth and sales of inexpensive calculators and their consequent widespread availability to pupils demand that the mathematics curriculum be reexamined and that teachers use calculators as an instructional tool. Still others, through the NACOME Report (Conference Board of The Mathematical Sciences 1975) and official policy statement by the National Council of Teachers of Mathematics, have encouraged schools to formulate calculator policies and to make these tools available to pupils to be used in creative and innovative ways of learning mathematics.

Pleas for information about calculator usage have been made at the

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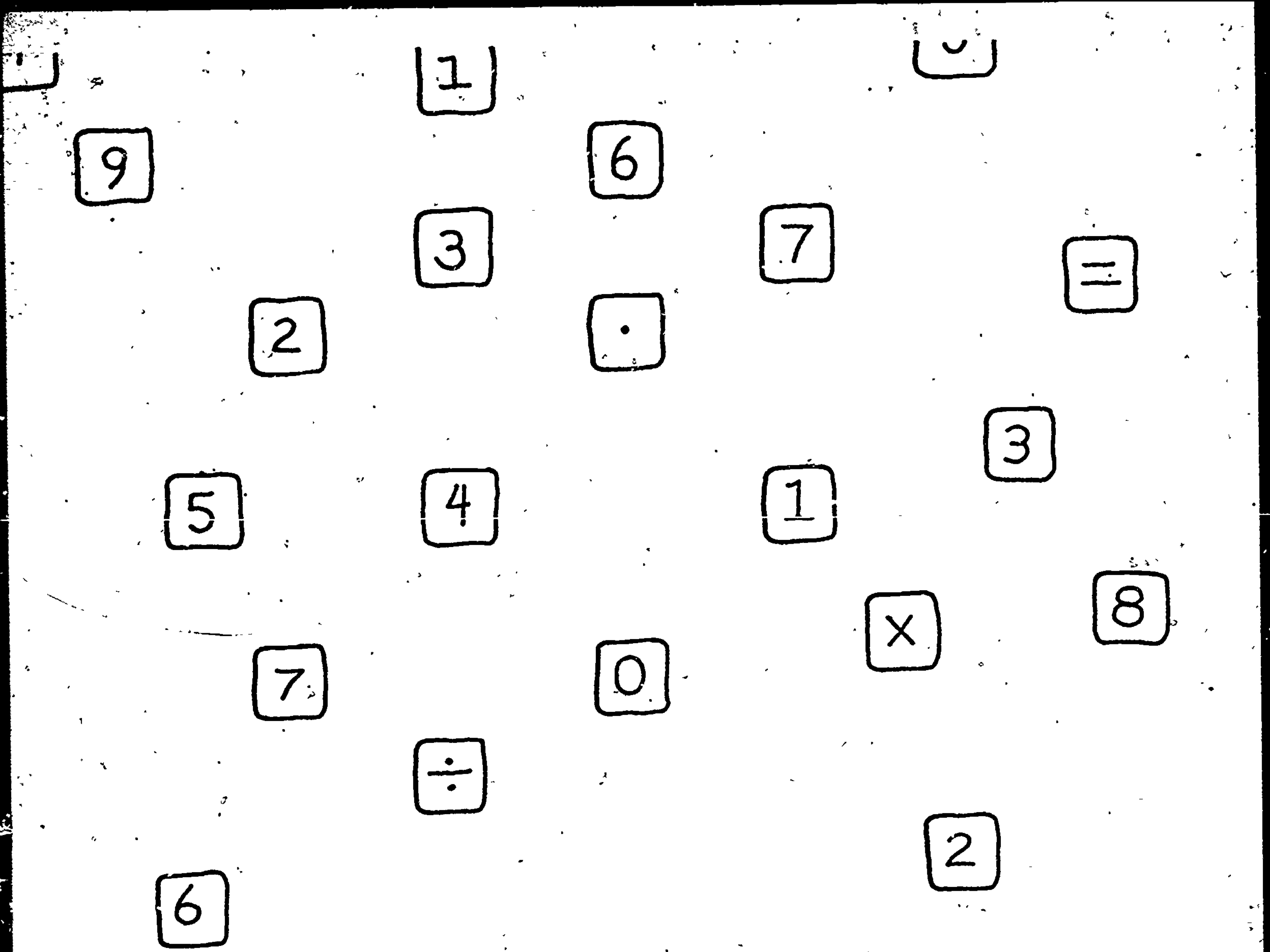


grassroots level by parents and teachers as well as by research groups—for example, the recent *Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics*. One of the recommendations from this conference was to survey current instructional policies to answer questions such as, How extensively are calculators being used in classrooms? It is toward that end that this research was conducted.

The Study

In the spring of 1979, a survey of classroom teachers in Missouri was conducted to check the current attitude toward and the extent of hand calculator usage. Rather than mailing questionnaires, which typically has less than an acceptable rate of return, it was decided to involve fewer teachers and to interview each of them individually in order to obtain more complete and accurate information. Ten school districts were randomly selected from the *1978–79 Missouri School Directory*. In order to insure the representation of a variety of school districts, two urban, three suburban, three small community, and two rural school districts were selected. The following criteria were used to characterize these four different types of school districts.

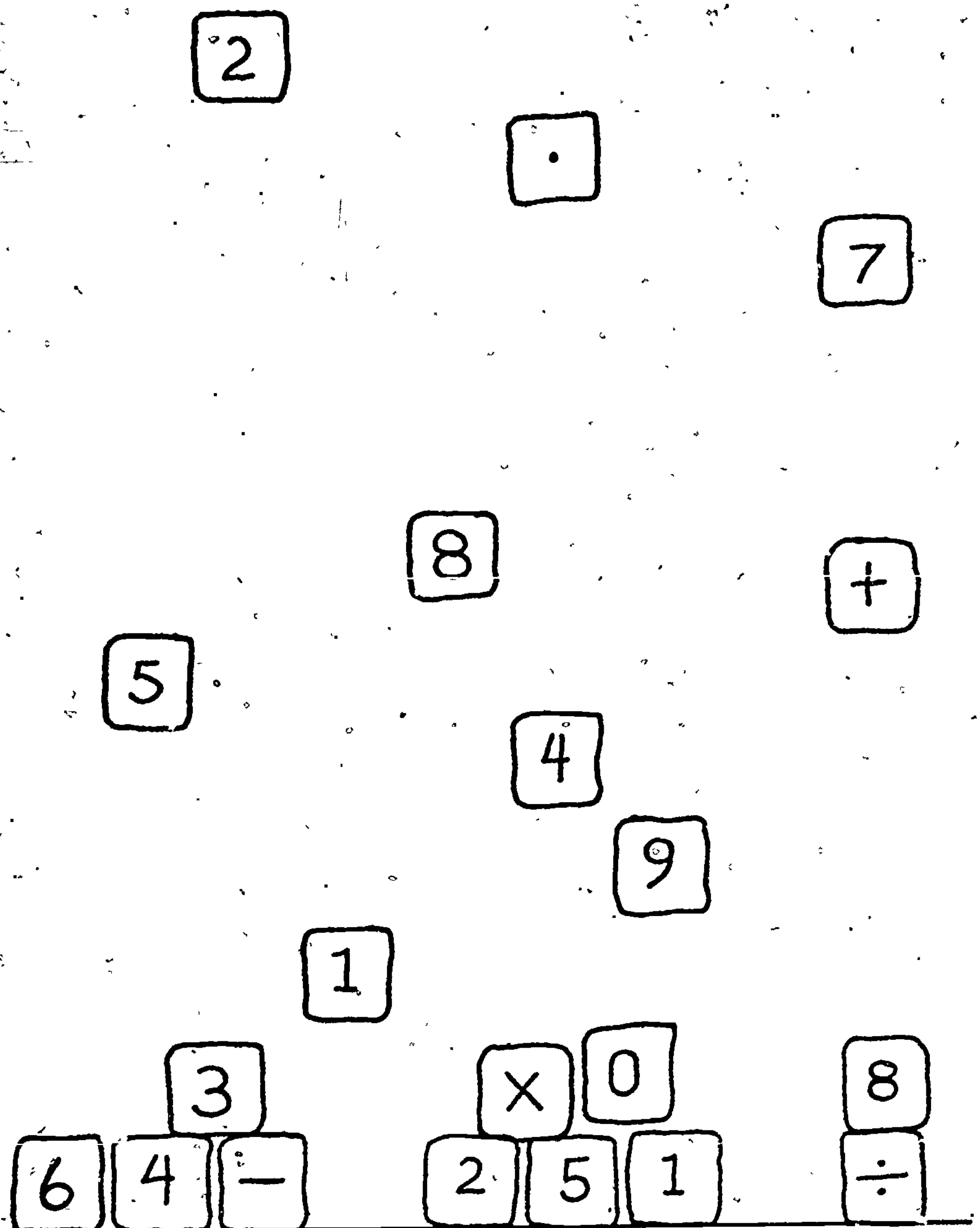
Urban: located in a metropolitan area with a population exceeding 125 000.

Suburban: located in a municipality adjacent to an urban area or within ten miles of an urban area.

Small Community: not suburban and servicing a population between 10 000 and 125 000.

Rural: not suburban and located in a town with a population less than 10 000.

Once a school district was selected, a second stage of random sampling was done to choose one elementary, one junior high, and one senior high school within the district. The superintendent of each district was contacted and asked for permission to contact principals within the selected schools. Every superintendent expressed keen interest in the survey and agreed to cooperate.



The third and final sampling stage involved the identification of classroom teachers. Every full-time mathematics teacher in the junior or senior high school was scheduled for an interview. Two elementary teachers at each grade level from grades 1 through 6 were randomly chosen to be interviewed.

This procedure was used to schedule interviews with 202 teachers in the ten districts. Eight teachers were absent when the interviews were conducted in their schools. It was not feasible to reschedule these interviews so the results of this survey are based on information gathered from 194 teachers. Thirteen percent of these teachers indicated they were members of the National Council of Teachers of Mathematics. Each interview followed a similar set of questions and required from 10 to 25 minutes to complete. The questions used in the interviews addressed key issues related to calculator usage. The protocols were developed with the help of local classroom teachers, mathematics coordinators from several school districts, and several mathematics educators.

Prior to the interviews, the teachers were asked to survey their classes to determine how many of the children had calculators available at home. Sixty-eight percent of the students reported having access to at least one calculator. This percentage varied little when looking at the data by grade level. These percentages are slightly below those found in the 1977–78 National Assessment of Educational Progress, which reported that over 75 percent of the nine-year-olds, 80 percent of the thirteen-year-olds, and 85 percent of the seventeen-year-olds had access to at least one calculator.

The opportunity to sit down with teachers individually, outside the classroom, allowed for gathering not only factual information, but also teacher insights, questions, and concerns regarding calculator usage in the schools. Highlighted here are several of the questions used in the interview, along with a description of the teachers' answers and comments.

Does your school (school district) have a calculator policy?

An overwhelming majority of the 194 teachers interviewed said no current calculator policy existed in their district. Seventeen teachers reported their school had a calculator policy, but the exact nature of this policy was never determined. The confusion among teachers regarding this question apparently stemmed from what they thought constituted a policy, whether it be written or perhaps what their principal or another teacher might have said to them informally. Some actual teacher comments illustrate this uncertainty:

> "I don't know if we have a policy but I have heard we are not supposed to use them."
>
> "I think our policy prohibits calculator use."
>
> "They can be used in the senior high but our board policy prohibits us from using them in the junior high."
>
> "Our principal doesn't want calculators used in mathematics classes."

After talking with the teachers, interviewers frequently contacted principals, curriculum coordinators, or district superintendents in an effort to track down specific policy. In no case were they able to locate a written policy formulated by a Board of Education, superintendent, principal, or mathematics department.

The interviews revealed that some schools are beginning to prepare a policy or identify some guidelines for calculator usage. If such a policy reflects teacher input, it will be very flexible. Of the teachers interviewed, none felt a

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school policy should unequivocally ban calculator usage and there were none who felt it should be required. The *overwhelming feeling was that calculators exist, that there are many appropriate places for using them at all levels of the mathematics curriculum, and that the type and extent of this usage should be left up to the discretion of the individual classroom teacher.*

This provides a background for the questions that follow and the specific statistics that are reported in table 1. Although percents are reported in table 1, color coding is also used to provide a visual interpretation of the results. Different shades, from light (0–20%) to dark (81–100%), suggest some trends and patterns. Statistics are reported for the whole group and for each grade level. In some cases, totals for a given question do not add to 100 percent. This is due to rounding and in part to some teachers not answering certain questions. A careful examination of table 1 will provide insight into interpreting the results of this survey.

Should calculators be available to children in school? If so, to which children? At what grade level?

Eighty-four percent of the teachers said calculators should be available to children in school. Many were careful to add that supervision by the teacher should be an important element. Table 1 shows a breakdown as these teachers characterized which children and at what grade level calculators should be used. There are certain trends indicated by these data. One interesting trend is that, in general, teachers are likely to say calculators should be used, but they should be used in higher grades. For example, often fourth-grade teachers would say calculators should be available, "probably in the upper grades and high school, but not in my grade." This feeling that "it's okay for everybody except my grade" may be evidence of the teachers' lack of confidence in using calculators as an instructional tool. Teachers were more favorable toward using calculators with all children rather than with only the best or poorest students. At the higher grade levels, however, it was common for teachers to comment about calculators being a time saving, more efficient computational device than paper-and-pencil calculations for the best students.

Teachers who had used calculators were asked if it had changed their emphasis on mathematics content. Many commented that not only could they work more problems if a calculator was available, but also they actually covered more topics. They also reported dealing more with concept development and less with computation during their mathematics class. Furthermore, more than 80 percent of them reported observing attitudinal changes in their students. Without exception these changes were positive and were characterized by teacher comments describing students as being eager to attack problems, showing greater confidence in ability to solve mathematics problems, and becoming more excited about doing mathematics.

Several teachers reported an interesting psychological problem resulting from calculator use, namely, convincing children that calculators are legitimate tools to use in a mathematics class. Some children felt it was cheating to use a calculator and expressed guilt feelings when they used them. These teachers said they worked hard to dispell this notion and to nurture the idea that use of calculators in mathematics class was perfectly acceptable.

Can children bring their calculators to your class and use them?

Forty-two percent of the teachers reported that children could bring calculators to their class and use them. Many teachers, however, said this question had never come up. Their pupils occasionally brought their calculators to school to show their friends or for play during free time, but they had never asked to use them during a mathematics lesson. Primary, senior high, and experienced teachers are most likely to allow calculators to be brought and used in their classroom. Whereas, table 1 reports far less intermediate and junior high teachers allowed children to bring and use calculators. Why? One explanation is the heavy emphasis on computational algorithms that are introduced and practiced and practiced and practiced . . . during this time. Use of calculators could greatly alter such a mathematics curriculum!

Have you used calculators in your math class? What for? Would you like to?

The most frequent use of calculators was reported at the senior high level, with little being done at the elementary level. Teachers who had used calculators reported using them most often with problem solving, word problems, and basic facts. These teachers indicated they had found the calculator useful for developing certain mathematical concepts as well as for computation.

Of those teachers who had not used calculators, forty-two percent said they would like to. By far, the most desire was expressed at the senior high level. Elementary teachers generally expressed caution, saying they would use calculators if they were convinced calculators were an appropriate instructional tool and if they could receive proper training. Several also indicated a concern for declining test scores if children used calculators.

Is special training needed to use calculators effectively?

Elementary and junior high teachers were clear in expressing the need for training. In talking with some teachers, it became apparent that they understood training to mean learning to operate their particular calculator. They had not thought of training in terms of learning ways to use the calculator effectively and incorporating it in their daily lessons as another instructional tool. Only 9 percent of the entire group—mostly senior high teachers—had attended a calculator workshop or training session, although 67 percent said they would like to attend such a session. Palmer (1978), in a survey of leadership personnel responsible for mathematics instruction found this same attitude: "Teachers felt that there was a definite need for workshops to help them develop and improve their competence in the use of calculators in mathematics instruction."

Teachers cited several reasons for their lack of use of calculators in the classroom. Primarily, inservice training was not available. They also mentioned that they saw little reason to get

Table 1: Percentage of Teachers of Mathematics Who Responded "Yes" to Selected Questions from a 1979 Calculator Usage Survey. (N = 194)

Legend: 0- 20% | 21- 40% | 41- 60% | 61- 80% | 81-100%

Question	Whole (N = 194)	Primary (N = 51)	Intermediate (N = 49)	Junior high (N = 48)	Senior high (N = 46)
Should calculators be available to children in school?	84	85	78	85	89
Which children:					
all*		88	55	68	68
best*		7	37	33	25
poorest*		5	8	0	8
What grade level:					
primary*		49	21	26	28
intermediate*		[illegible]	[illegible]	36	43
junior high*		69	[illegible]	69	63
senior high*		[illegible]	97	100	100
Can children bring their calculator to your class and use them?	42	47	27	29	65
Have you used calculators in your mathematics class?	35	14	23	42	62
Would you like to?*		44	35	34	62
Is special training needed to use calculators effectively?	65	84	78	63	33
Have you attended a calculator workshop?	9	2	6	13	17
Would you like to attend a (another) calculator workshop?	[illegible]	[illegible]	78	74	63
Should children master the four operations of arithmetic before they use calculators?	[illegible]	76	74	83	89
Does use of calculators cause children to lose ability to compute or to remember basic facts?	43	37	49	44	41
Would you support calculator use on standardized tests measuring concepts or applications?	43	35	18	52	67
Does your mathematics textbook have activities written for calculator usage?	11	6	10	10	20
Should your text have activities for calculator usage?	51	35	52	55	65

*Reports percent of teachers who responded yes to this question and the preceding one.

training in this area when no calculators were available for use in their classrooms. Although about 35 percent of the teachers reported calculators were available for classroom use, further examination showed this typically meant a few calculators were available in a mathematics lab or Title I mathematics classroom, or perhaps a set of calculators was available on a check-out system. Rarely were there sufficient quantities to provide a calculator for every student or even for every pair of students in a classroom. Often it was found that teachers within a school where calculators were available were unaware of their existence. In no instance were there sufficient quantities of calculators available to provide for two or more teachers who wanted to use them at the same time.

When asked what specific information or material should be included in a calculator training session, the teachers identified the following as high priorities: sharing of specific instructional activities and lessons that use calculators; discussions with teachers who had used calculators and who would share their perceptions and experiences; learning of effective ways to communicate with parents regarding the role and value of calculators in a mathematics program; and reporting of recent research related to calculator usage.

Should children master the four operations of arithmetic before they use calculators?

Eighty percent of the teachers felt children should master this material prior to using calculators. Several indicated that if this was not done, the calculator would be a crutch and children would have little incentive to learn to compute. Other teachers, however, said some of these skills could be developed with the calculator. One teacher remarked that if students could not compute, they probably would not be able to use a calculator effectively either.

Teachers generally agreed that with slow students or students at the senior high level who had never learned to compute, a calculator should be used because these students would probably never learn to compute otherwise. This comment is in the same spirit as a recommendation found in the *Overview and Analysis of the School Mathematics Curriculum, K-12* report, although none of the teachers interviewed mentioned this NACOME Report. None of the junior high teachers reported that calculators should be made available to their poorer students, which is in direct conflict with the NACOME recommendation "that beginning no later than the end of the eighth grade, a calculator should be available for each mathematics student during each mathematics class. Each student should be permitted to use the calculator during all of his or her mathematical work including tests."

Does use of a calculator cause children to lose the ability to compute or to remember basic facts?

Few teachers said they could answer this from first hand experience, but 43 percent felt that using a calculator would cause students' ability to compute to decline. Twenty-two percent said they were not sure but would like to know. The ambivalency of the responses to this question reveals the need for dissemination of recent research on calculator usage. One of the common findings among such studies is that children with calculators available during mathematics classes do as well on traditional paper-and-pencil computation tests as classes who have never used calculators. (Wheatley, et al 1979)

If a calculator is made available to each student, would you support calculator use on standardized tests measuring concepts or applications?

When this question was posed, teachers were reminded that most standardized tests have a computation subtest along with portions measuring applications, concepts, or problem solving: and furthermore, their answers should be directed toward the parts of the test not dealing exclusively with computation. Over 40 percent of all teachers said yes to this question and supported their position with statements such as the following:

> "This would provide a better measure of what they know."
>
> "Calculators are not going to give them any answers, they will do only what they are told to do."
>
> "Calculators are a part of the real world and should be used."
>
> "Calculators are a tool—like a pencil—and should be available."
>
> "Calculators allow more time to concentrate on the process."

On the other hand, teachers who felt calculators should not be used on these tests commented as follows:

> "How will we know what they know?"
>
> "Standardized tests should reflect what students know without a calculator."
>
> "Using calculators is not fair, students must do their own thinking."
>
> "I want students to have the skills I learned."
>
> "It's like digital watches, kids won't know how to tell time any more."

Does your mathematics textbook have activities written for calculator usage?

A review of current textbooks reveals few activities explicitly written for calculator usage. The interviews confirm this: only 11 percent of the teachers said their textbooks had such activities and most of those were senior high teachers. Over half of the teachers said the textbooks should include instructional activities providing for calculator usage. Several teachers felt that it should be their responsibility to adapt textbook activities for calculator use, but the overwhelming majority indicated that problems taking advantage of the calculator's capability as well as instructional suggestions should be included in their mathematics textbooks. Some teachers also stated that such activities should be an integral part of the content being learned and not simply tangential problems, challenges, or artificial uses for the calculator.

Summary

This statewide survey of calculator usage in Missouri schools provides much food for thought. Here are four of the juciest morsels to chew on:

1. Sixty-eight percent of students, grades 1 through 12, have at least one calculator available at home.

2. Eighty-five percent of all teachers said calculators should be available to students in school.

3. Almost two-thirds of the teachers said that special training was needed to use calculators effectively.

4. Over half of the teachers would like to have calculator activities integrated within their regular mathematics textbook.

There are many other interesting findings but these four document that teachers perceive the need for calculators within the school mathematics program as well as their own need for inservice training with them. Furthermore, a sizeable portion would like calculator activities included within their regular mathematics program. Taken collectively, these morsels as well as the overall results from this calculator usage survey suggest that things are changing; but much more leadership, direction, and training is sorely needed if calculators are to be used to their fullest potential.

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CALCULATORS IN THE ELEMENTARY SCHOOL:
A SURVEY OF HOW AND WHY

by

Leonard E. Etlinger and Earl J. Ogletree

The availability and decreasing cost of hand-held calculators has put pressure on educators to carefully consider the issues related to classroom utilization of these devices. During a series of National Science Foundation-funded activities* in Chicago and St. Louis from 1978-1980, information was collected from more than 400 participants with regards to how and why calculators should be used in the classroom. The objectives of the sessions were information spread and exchange, and facilitation of decision-making: the participants included teachers, administrators, and parents.

Opinions on the issues appeared to have a common consideration; i.e., implementation of hand-held calculator programs should be based on sound policy, well-defined procedures, and the caution due any innovative activities in education. The participants agreed that teachers need to be aware of how, when, with whom, and under what conditions calculators can be used in the classroom. They felt that calculators should be usd to reinforce basic facts, discover new concepts, explore number patterns, play instructional games, solve applicable problems, and aid in a wide variety of math, science, and other subject-matter learning. The only significant caveat developed by the participants was that

> Calculators, like any educational innovations--whether ideas, materials, or devices--can be misused in the classroom. With calculators such misuse could be highly detrimental to the students' arithmetical development.

Findings

The participants first responded to a variety of questions in discussion, debate, or panel response formats and later formed working groups for the development of recommendations. Responses to selected questions and issues are presented next.

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CALCULATORS IN THE ELEMENTARY SCHOOL: A SURVEY OF HOW AND WHY

Can Children Learn Mathematics From Using Calculators In School?

Responses were almost entirely positive. Some typical responses were:

Children can explore the ideas behind arithmetic, concepts of mathematics, numerical patterns, very large and very small numbers, etc.

Calculators can help with basic facts, multiplication and division, order of magnitude, intuitive concepts, and motivation in math.

Children will learn more because it will be more interesting and fun.

Kids can work with very large numbers and experiment with a variety of processes such as the square and root of numbers.

Calculators can reinforce place value and number sequence. Approximation techniques can be developed.

Should Students Be Permitted To Use A Calculator All Of The Time In School?

Probably Not! Children need to learn facts and basic skills without calculators.

Math activities should be planned specifically for calculator or noncalculator usage. Attention should be given to the objectives of the activities.

Children in junior and senior high school should probably be allowed to use calculators in most of their school work and tests.

Very young children (kindergarten and first grade) should only use calculators in a few carefully designed instances.

Can Calculators Facilitate Creative Thinking In Students?

Most responses were affirmative. It was felt that calculators could enhance creativity

There are many fine examples of activities where children use their own creativity together with the calculator in numerical exploration.

Pencil and paper calculation, particularly when not mastered properly, restricts creativity.

Students are not limited to "nice" or "easy" numbers. Decimals, nonintegral quotients, large numbers, and negative numbers can more easily be used in problems which are to be solved by calculators.

Some students will attempt difficult problems which they would not think of trying without calculators.

The gifted or advanced student will be challenged to explore sophisticated problems and concepts.

Should Children Use Calculators Before They Have Mastered The Basic Facts?

Children must, eventually, be able to perform basic arithmetic without calculators.

Calculators are motivational; this motivation may result in kids learning basic skills.

Children should still learn basic skills without calculators, for the most part, and use calculators for other types of math learning. This other math learning could, in some instances, take place before basic skills are mastered.

Possibly, older kids who have still not mastered basic skills can benefit by moving on in math and science with the aid of the calculator.

Should Calculators Be Used On Tests?

Clearly not on tests of basic arithmetic skills.

They probably should be allowed on most higher level tests.

Tests should reflect real life needs. When appropriate, test rules should be changed to allow the use of calculators. National and regional tests, including standardized achievement tests, college admissions tests, and vocational aptitude tests, should be reviewed in light of this changing technological aspect of society.

The project participants also generally agreed on the following points:

- Calculator activities must be carefully designed if genuine learning is to take place.

- Teacher training will be critical to proper school utilization of calculators.

- Many math activities, particularly in the primary grades, must still be done <u>without</u> calculators.

- The greatest threat posed by calculators is that of <u>misuse</u> in the classroom. Poor teachers might use calculators as an excuse to avoid real teaching and learning.

- Continued exploration, research, curriculum development, and policy formation is needed.

Recommendations

In a variety of working groups the project participants developed the following recommendations and statements concerning school utilization of calculators.

1. Primary Grades (K-3)

   A. Recommend usage of calculators to:

      1. Develop an understanding of the number system

         (a) provide additional practice with number sequencing and number recognition

         (b) develop place-value concepts

         (c) provide additional drill with basic facts

         (d) become more familiar with basic operations performed on numbers and number patterns

         (e) to aid in introducing the concept of fractions

         (f) to help the student become proficient in estimation

         (g) reinforce the base ten system (i.e., metric, money)

      2. To concentrate on problem-solving skills

      3. To promote success in arithmetic for slow learners

      4. To challenge the gifted student

2. Middle Grades (4-6)

   A. Reinforce and follow-up of ideas in grades (K-3)

   B. Develop objectives for hand-held calculators in the curriculum at each grade level

C. Each grade level and/or department should

1. Integrate calculators into the curriculum by means of various topics, such as

   (a) Science - interpreting data
   - measurement
   - metrics

   (b) Math - money, estimation, averages, place value, story problems

   (c) Economics - consumer education

   (d) Social Studies - map skills.

2. Utilize calculator activities that correlate with the scope and sequence of their math texts

3. Grades (7-12)

Uses - In review for preparation for high school placement tests

- To reinforce fractions, decimals, and related patterns
- Review concepts rather than tedious calculations
- Formula work - quick calculation (e.g., areas and volumes)
- To promote creativity in students to develop their own games and problems
- Creating math lab activities to reinforce concepts
- Solve statistical problems and their application
- Use to aid in the solution of real life problems
- To check answers and formulae of algebraic problems
- Introduction of programmable calculators to calculate commissions, mortgages, taxes, and interest.

The following recommendations were made regarding teacher training.

Staff Development

Staff development program should be sufficient to meet the objectives established for each of the grade and/or department levels: Primary, Intermediate, Junior High, Senior High.

Suggestions include:

1. In-service workshops

2. In-service credit or payment for courses

3. Funds for resource people, professionals, video-taped lessons

4. Released time to observe programs functioning

Facilities (beyond those available through an individual school district) for providing in-service training should be:

(a) University and college extension programs

(b) University and college short courses

(c) Facilities such as a learning center or teachers center. The teacher centers should provide facilities for staff development in the use of hand-held calculators.

5. Schools should train their own teachers to use calculators proficiently and have one "very proficient," or troubleshooter teacher, to help out with teacher problems; could be another professional, or a resource, to help with teaching problems.

6. Faculty must be supportive so there is no one undermining the program. School personnel (and parents) must be convinced.

7. Some guidelines should be established on use of calculators.

In addition to these suggestions additional research should be done to discover the "best" use of calculators in teaching and learning.

The participants in the workshop and survey were asked to list materials needed to implement and develop a calculator program in their school.

Materials

- Supply enough calculators for largest class in school (It is probably important that this set consists of one standard model)

- Booklet with suggested outline format including conceptual and skill activities appropriate to grade level

- Individualized packet for student enrichment

- Teacher packets that can be used in classroom situations

- Identification of the use of the calculator in subject matters other than math and science; setting up packets to be used
- Identification of the model of calculator used
- Identification of the updated resource materials available
- Learning Center activity books

This document may be obtained from EDRS as ED 191 741.

Using Electronic Calculators With Third And Fourth Graders: A Feasibility Study*

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During the summer of 1974, a sequence of mini-experiments was conducted by the authors, exploring ways to use electronic calculators with children ages 5-7. The results showed that the children responded enthusiastically to the calculator, and that the students who used them showed considerable gains in mathematics achievement.

As a sequel to this sequence of mini-experiments, a small experiment was run using the hand held calculator with children ages 7-9. The experiment was designed to help answer these questions for grades 3 and 4: (1) which standard mathematics topics can be taught most effectively using the hand held calculator? (2) What implications does the hand held calculator have for problem solving situations? (3) What new mathematical topics can be successfully introduced because of the availability of the electronic calculator? For comparative purposes, a standardized mathematics achievement test was administered both before and after the instruction.

Five third and fourth grade children participated in the study during a ten week period in the fall of 1974. Three of the children attended regularly and were given the Metropolitan Achievement Test, both before and after the instruction. The other two attended irregularly, and were not tested. Instruction took place during 32 class meetings during the 10 week period, with each meeting lasting 30 to 60 minutes.

Among the topics which were introduced were the following:

Operation of the calculator.**

Using large numbers on the calculator. The children discovered that some large numbers, such as 180 billion, cannot be shown on the calculator.

Writing numbers using only the 1, 0, + and = keys

Negative numbers (introduced using temperature and a number line with spaces to the left of 0); addition of signed numbers.

Internal logic of the hand held calculator; discussed visual appearance and unseen memory of the calculator as a problem is entered.

Estimating: getting "ball park" answers for addition and subtraction problems, then using the calculator to check the estimate.

* This research was supported by Texas Instruments, Inc. working under contract with the MERGE Institute.

** Texas Instruments Model TI 2500 was used throughout the study.

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Number tricks (take a number with 3 different digits, reverse the digits, subtract the smaller from the larger, the sum of the difference and the difference with the digits reversed is 1089)

Area

Decimals (introduction to tenths and hundreths using a square divided into 100 parts; addition, subtraction, and multiplication of decimals)

Decimal-fraction equivalencies

Inequalities with decimals

Unit pricing

Flow charts for adding and multiplying using the hand held calculator.

Prime and composite numbers; prime factorization of numbers up to 50.

Throughout the ten week period, a great deal of time was spent reviewing and practicing the arithmetic operations with whole numbers and decimals.

For the three children who attended regularly, the results on the Metropolitan Achievement Test (reported by grade level) were as follows:

		Before Instruction	After Instruction	Gain	
	Computation	4.3	6.2	+ 1 yr.	9 mo.
A	Concepts	7.3	7.7	+	4 mo.
(age 9)	Problem Solving	7.0	7.0		
	Total	5.6	6.9	+ 1 yr.	3 mo.
	Computation	3.8	5.5	+ 1 yr.	7 mo.
B	Concepts	3.8	5.6	+ 1 yr.	8 mo.
(age 7)	Problem Solving	3.7	4.9	+ 1 yr.	2 mo.
	Total	3.6	5.6	+ 2 yr.	
	Computation	2.4	2.9	+	5 mo.
C	Concepts	3.8	4.9	+ 1 yr.	1 mo.
(age 9)	Problem Solving	3.4	4.4	+ 1 yr.	
	Total	3.0	3.7	+	7 mo.

While these large gains in mathematical ability are quite impressive, several limitations must be mentioned. Small group, essentially individualized instruction, is not typical of the classroom teaching-learning environment. In a one-to-one situation, a skilled and careful teacher can immediately correct any misconception, and adjust work level to student needs. For example, one child was given a quick review of subtraction of whole numbers when he experienced difficulty in subtracting decimals. Also, the Metropolitan Achievement Test relies heavily on accuracy in addition and multiplication tables, and much time was given to practive on tables, albeit with the calculator.

However, these facts are not enough to entirely discount the very real

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gains that were shown. Several of the children were in the upper range of the scale originally, and the retest did not necessarily represent a real power test for them. Gains in areas such as understanding of negative numbers, comprehension of flow charts, real world problem solving skills, and actual ability to compute with the hand held calculator were not measured at all. Nor will they be measured in other standardized tests generally in use.

SUMMARY

As a result of this study, we would like to offer these tentative answers to the three questions asked about third and fourth grade mathematics.

Question 1: Which standard mathematical topics can be taught most effectively using the hand held calculator?

a) The hand held calculator is especially useful for testing and practicing place value skills with whole numbers and decimals. Quick, non-tiring practice (as contrasted with written practice) can be given.

b) Negative numbers are discovered through just "playing" with the hand held calculator. Formulas such as $A < B \Longrightarrow A - B = {}^{-}C$ can be tried with many numbers so the child can see the pattern.

c) Decimal and metric measures, including area and volume, can easily be taught.

d) Prime and composite numbers can be more easily identified because of the capability of quickly and accurately checking factors.

e) The calculator is especially good for going from fractions to decimals, but difficult the other way.

f) In general, a guided discovery method can be used because the children can quickly try many examples and detect patterns and algorithms.

Question 2: What implications does the calculator hold for problem solving situations?

a) The students are able to obtain much more practice with "real world" verbal problems (which are closer to real life than typical verbal problems are). The children do not have to write the equation, which can be tiring, nor do they have to remember the numbers and operations mentally.

b) One surprising benefit of using the hand held calculator in a verbal problem solving situation was a test of how quickly different children process oral information. A teacher can readily detect children with weak listening skills and weak short-term memories by taking note of children who continually need the problems repeated. In fact, practice with this type of problem and increasingly longer and more difficult examples seemed to help the children gain speed in listening skills.

c) The children enjoyed and were highly motivated to try fairly complex real world problems involving several operations. Teachers used the calculator to explore large number problems that occurred in other subject areas such as social studies and language arts. Students made up their own problems and tried solutions, often discovering a need for operations or algorithms which hadn't yet been taught.

d) Because the computation is done easily, the emphasis in teaching can be placed on problem definition, delineation of relevant and irrelevant information, operations involved, formula and equation writing, pattern recognition, and algorithm formulation.

Question 3: What new mathematics topics can be successfully introduced because of the availability of the electronic calculator?

a) In general, from the brief experience in this study (grades 3-4) and discussion with teachers, the standard mathematics curriculum can be expanded in computation to include use of numbers of greater magnitude. A shift of emphasis occurs, so that estimating skills, the use of negative numbers, and decimals occur at a much earlier time than normally taught in the standard mathematics curriculum. In other areas, such as pre-computer skills, the use of flow charts and discovery of "debugging" techniques become an intrinsic part of the curriculum.

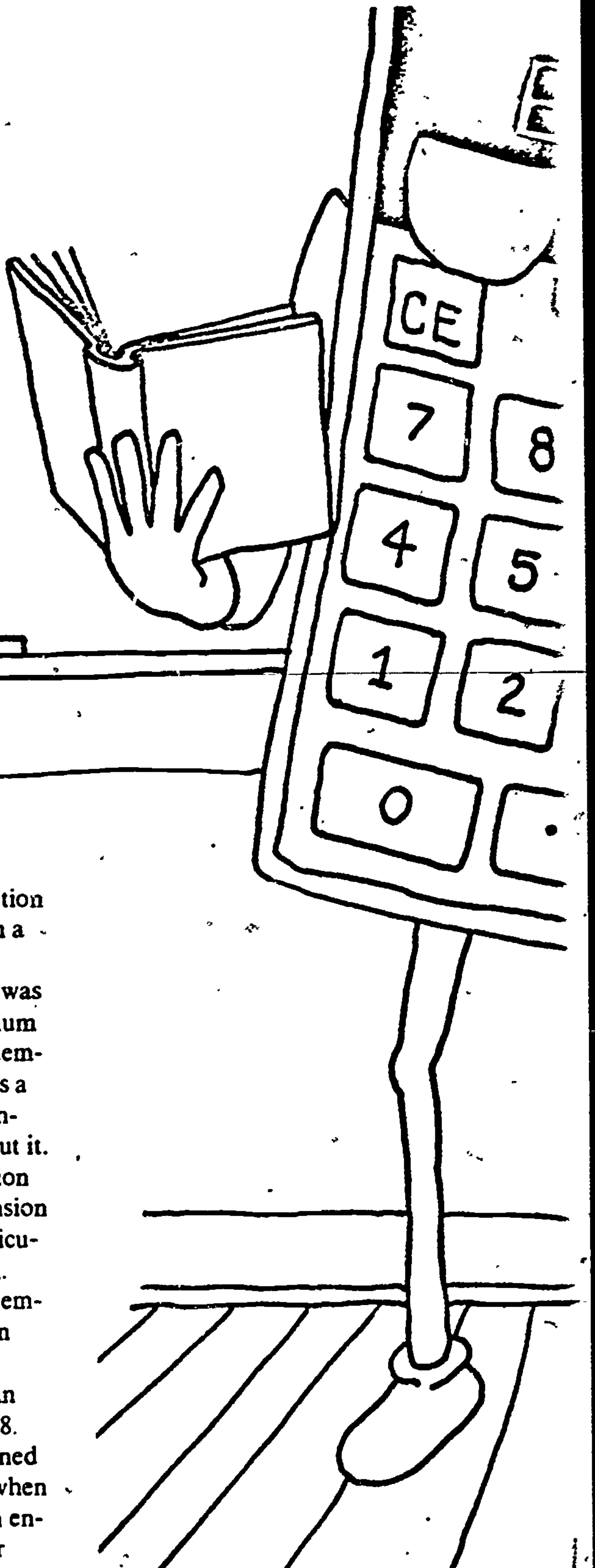
b) The teacher has an opportunity to spend more time on concrete representations of concepts since she can check instantly for student understanding by having the children show answers on the calculator. Patterns can be more easily detected and explored. The children are not tied to a writing surface when they explore mathematics.

c) The teachers expressed a desire for problems using large numbers. As one teacher said, "a book of 'fantastic' large number word problems is needed. Things kids can relate to, such as large number attendance figures as parades, fairs, or numbers of hamburgers or pizzas eaten at a restaurant."

Finally, we should comment on one concern expressed to us by teachers: How will students who have used the calculator do on standardized tests? Our strong, but tentative results should help in this regard. In this study we incorporated many activities using the calculator to "drill" the children in addition combinations and times tables. These activities could easily be incorporated into any calculator curriculum.

The Calculator in the Classroom

By Norma Zakariya, Margo McClung, and Alice-Ann Winner



The hand-held calculator was introduced into the mathematics curriculum of the United Nations International School at the upper-elementary level during the 1977–78 school year. The new program was inspired by a summer workshop at Teachers' College, Columbia University. Calculator instruction began on an experimental level in a fifth-grade class in November 1977. This was followed by instruction in a fourth-grade class, and later in a third-grade class.

The goal at the fifth-grade level was enrichment of the existing curriculum and instruction was organized to demonstrate the use of the calculator as a learning tool for exploration of concepts too difficult to execute without it. The main direction of the instruction at the fourth-grade level was extension and expansion of the existing curriculum, especially in problem solving. Lessons were developed, also, to demonstrate the use of the calculator in real-life situations.

The experiment was not begun in the third grade until February 1978. This was not by design, but happened to be the point in the school year when the experiment had already shown encouraging results in the two higher grades. It did prove, however, to be the optimum time to begin; the children

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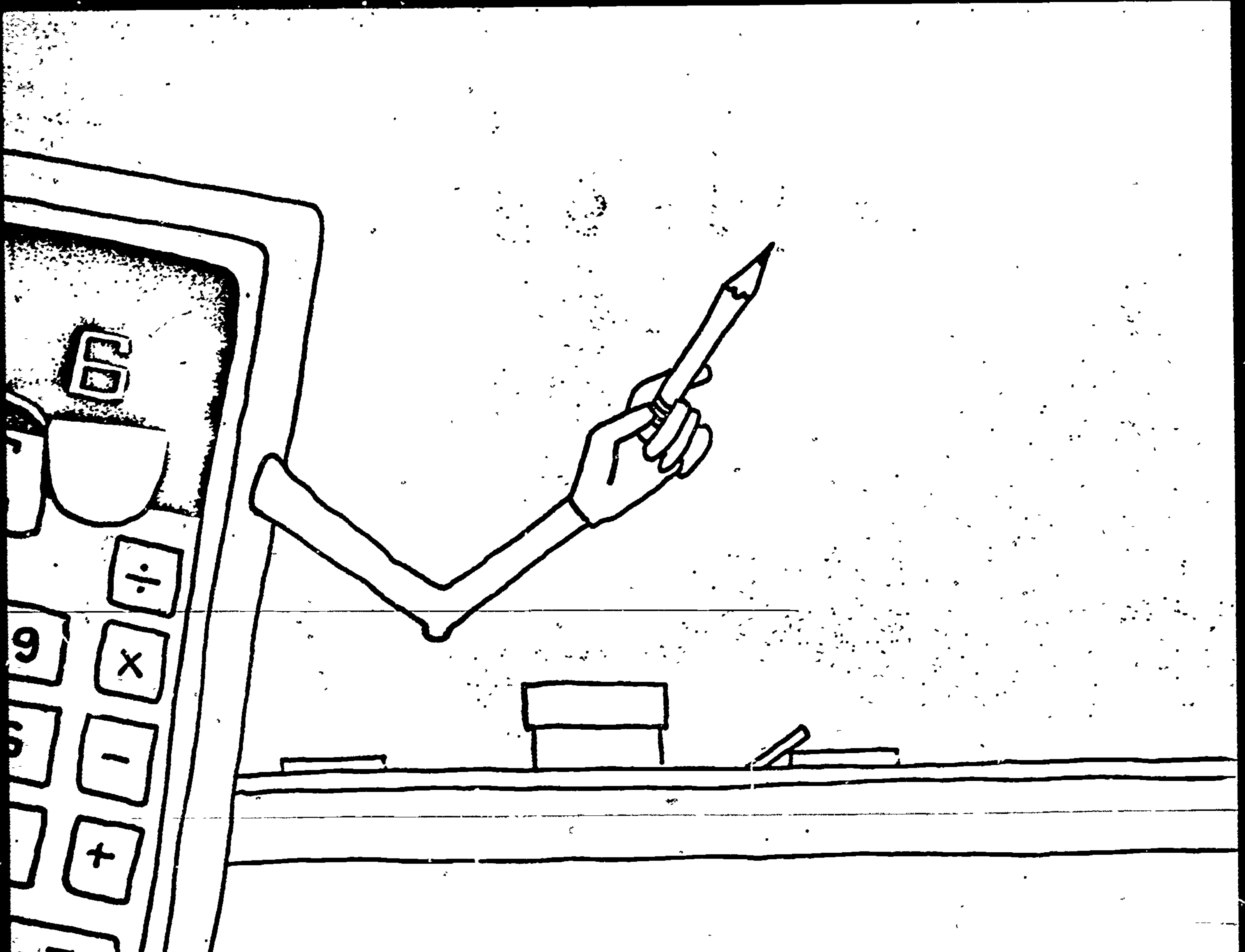
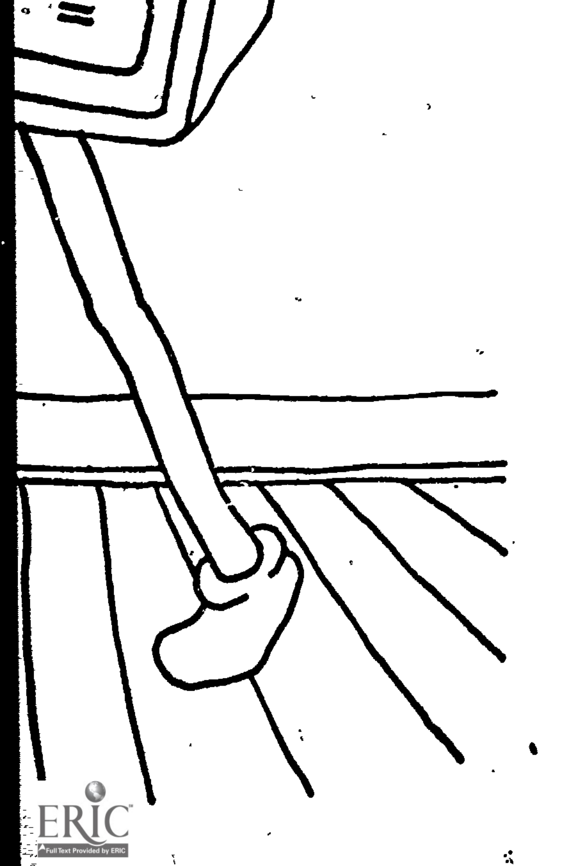




had reached a maturation level in their skill development that enabled them to profit from calculator instruction. The instructional emphasis was varied: reinforcement of basic concepts, development of number sense, problem solving, and a greater appreciation of a mathematical system.

The parents were informed of the program and the rationale behind it either at the open-house at the beginning of the school year or by letter prior to the start of instruction. The solid support of the administration and the enthusiasm of the children greatly contributed to the success of the experiment.

Three examples of lessons taught in the various classes are described here. These particular lessons are reproduced in this article to illustrate the diversity of the program and not because they are necessarily the best examples of the many lessons prepared.

Third Grade

Subtraction with regrouping

Third grade children often have difficulty in understanding the concept of place value. Subtraction with regrouping is, therefore, a process which they frequently find hard to master. The lesson in figure 1 was designed to help them overcome their problems in these two important areas.

The steps were as follows:

1. The child decides if regrouping is necessary.
2. If so, the child regroups without the calculator.
3. The child then enters the elements of the regrouped number on the calculator to check that the regrouped number does equal the original number.
4. The child proceeds to subtract.

We found that those children who had been unable to cope with the concept prior to instruction with the calculator were able to do so upon completion of these exercises. Even children who had been successful before the calculator was used found this lesson fascinating and developed a clearer understanding of the operation. We felt that the lesson not only facilitated the teaching of the subtraction-with-regrouping concept, but also increased the children's basic number sense.

Fourth Grade

Estimation

Fourth-grade children frequently have difficulty understanding the concept of estimation and the value of this process. The lesson in figure 2 was designed to demonstrate that estimation is a usable, everyday process. It was planned to—

(1) broaden the understanding of estimation;

(2) give an added dimension to the use of estimation;

(3) show how estimation may be applied to everyday life.

The children were asked to bring a grocery advertisement from a newspaper or a flyer from a grocery store.

The children found that the calculator freed their minds to concentrate on developing a method of spending the money within certain limits. In problem D, the children faced a new situation and found the calculator invaluable in finding the cost per ounce and finding the better buy. The exercise increased the pupils' understanding of grocery shopping and the use of the calculator in an everyday situation. They thought about the problem to be solved and not just the arithmetic.

Fifth Grade

Number theory: Quadratics

In this introduction to quadratic equations, the goals were to—

(1) extend mathematics knowledge by exploring new mathematical ideas;

Fig. 1

Regrouping

Copy the examples and write the missing numeral.
Use your calculator to check your work.

For example:	h	t	o
988 =	8	18	0

Find the sum of 800, 180 (18 tens), and 8 ones. The sum 98
988 will appear, which is the number you regrouped.

1. *h*	*t*	*o*	2. *h*	*t*	*o*	3. *h*	*t*	*o*	4. *h*	*t*	*o*
7	8	5	6	4	3	4	0	2	3	9	7
?	18	5	?	14	3	?	10	2	2	?	7

5. *h*	*t*	*o*	6. *h*	*t*	*o*	7. *h*	*t*	*o*	8. *h*	*t*	*o*
2	5	1	9	0	7	5	3	2	3	1	6
2	4	?	?	10	7	5	2	?	2	?	6

Subtraction (Regrouping When Necessary)

Do the following subtraction examples

1.	260 −130	2.	160 −70	3.	87 −36	4.	89 −43
5.	106 −12	6.	144 −63	7.	129 −82	8.	143 −57
9.	474 −187	10.	146 −107	11.	243 −137	12.	243 −162

Now use your calculator to check your answers. If you made an error, do you know what you did incorrectly? Did you—

(1) Regroup incorrectly?

(2) Subtract incorrectly?

Try not to make the same error again.

(2) explore alternate methods of problem solving;

(3) introduce, at a basic level, an algebraic concept as a transitional experience preparatory to formal instruction in the middle school.

The students were told that the exercises would help them extend their knowledge of mathematics by exploring new ideas. They were given the examples in figure 3 with the information that the numbers that were needed to make each sentence true formed a type of pattern. They were challenged to find the pattern, and asked if there was more than one pattern.

Conclusions

Questions have been raised in the recent literature concerning the best use of the calculator in the elementary classroom. Should it be used as a tool of the existing curriculum or should the curriculum be changed to implement the calculator? Both uses were attempted at U.N.I.S. The third-grade lesson on regrouping demonstrates how the calculator can be used effectively to promote better understanding of a textbook lesson involving a concept that many children find difficult. The fourth-grade lesson on estimation goes one step beyond this approach. A

Fig. 2

A. List the costs of six items on your grocery list in the column marked "real." Round this figure to the nearest ten (or hundred) and list the rounded figure in the column marked "estimate." Then find the real total of your grocery list and the estimated total.

	Real	*Estimate*
1.	______	______
2.	______	______
3.	______	______
4.	______	______
5.	______	______
6.	______	______
Total	______	______

What was the difference between the real and estimated totals? ______

B. If you gave the grocer $30, would you receive change? ______

C. You are in the grocery with $10. From your grocery list, estimate the cost of eight items that you could buy with your $10 and have the least amount of money left over.

	Item	*Estimate*
1.	______	______
2.	______	______
3.	______	______
4.	______	______
5.	______	______
6.	______	______
7.	______	______
8.	______	Total ______

D. You wish to buy an item that is sold in two sizes:

Size 1	240 g	@ $1.09
Size 2	480 g	@ $1.79

Without using your calculator, which one is the better buy per gram?
With the calculator, how much does a gram cost in each size?

Size 1 ______ Size 2 ______

Which size would be the better buy? ______ How much would your savings per gram be? ______

textbook topic was selected, but the lesson was made more challenging. The calculator relieved the children of the burdensome calculations that more difficult new material involves. The fifth-grade lesson illustrates how the calculator makes it possible to go beyond the curriculum, studying a topic not normally included in a fifth-grade textbook.

New dimension

Concepts once thought too difficult for a particular age group were explored and understood with the help of this new tool. The children seemed to develope a much better number sense. Problems that the vast majority of the children were unable to solve without the calculator became manageable for all. It was possible to concentrate on analyzing *how* to solve the problem once the calculator relieved the children of the cumbersome computation involved in arriving at the solution. In short, the calculator was *not* simply used within the limitations of the textbook. Its use compelled the teacher to rethink traditional ideas and added a whole new dimension to the teaching of mathematics.

Increased understanding

Many children elected to work in pairs rather than individually. Invariably this promoted discussion and increased their understanding of the ideas and processes in their specific assignments, which, in turn, meant an increased level of interest and success. The work

Fig. 3

For each number sentence there is a number that will make the sentence true. Find a number for each sentence. Do you see a pattern in the numbers?

1. (___ × ___) − (5 × ___) + 6 = 0
2. (___ × ___) − (8 × ___) + 15 = 0
3. (___ × ___) − (15 × ___) + 50 = 0
4. (___ × ___) − (13 × ___) + 22 = 0
5. (___ × ___) − (102 × ___) + 200 = 0
6. (___ × ___) − (37 × ___) + 70 = 0
7. (___ × ___) − (108 × ___) + 800 = 0
8. (___ × ___) − (7 × ___) + 10 = 0
9. (___ × ___) − (28 × ___) + 75 = 0
10. (___ × ___) − (16 × ___) + 55 = 0

with calculators also established that those children with a good visual memory found it easier to memorize computation facts with the continued use of the calculator.

Exploration and discovery

Some of the calculator lessons were designed so that the children were compelled to explore and discover. In a lesson on number theory, for example, they were first asked to try certain examples *with* the calculator. They then had to solve similar examples, applying what they had discovered, *without* using the calculator.

Expansion

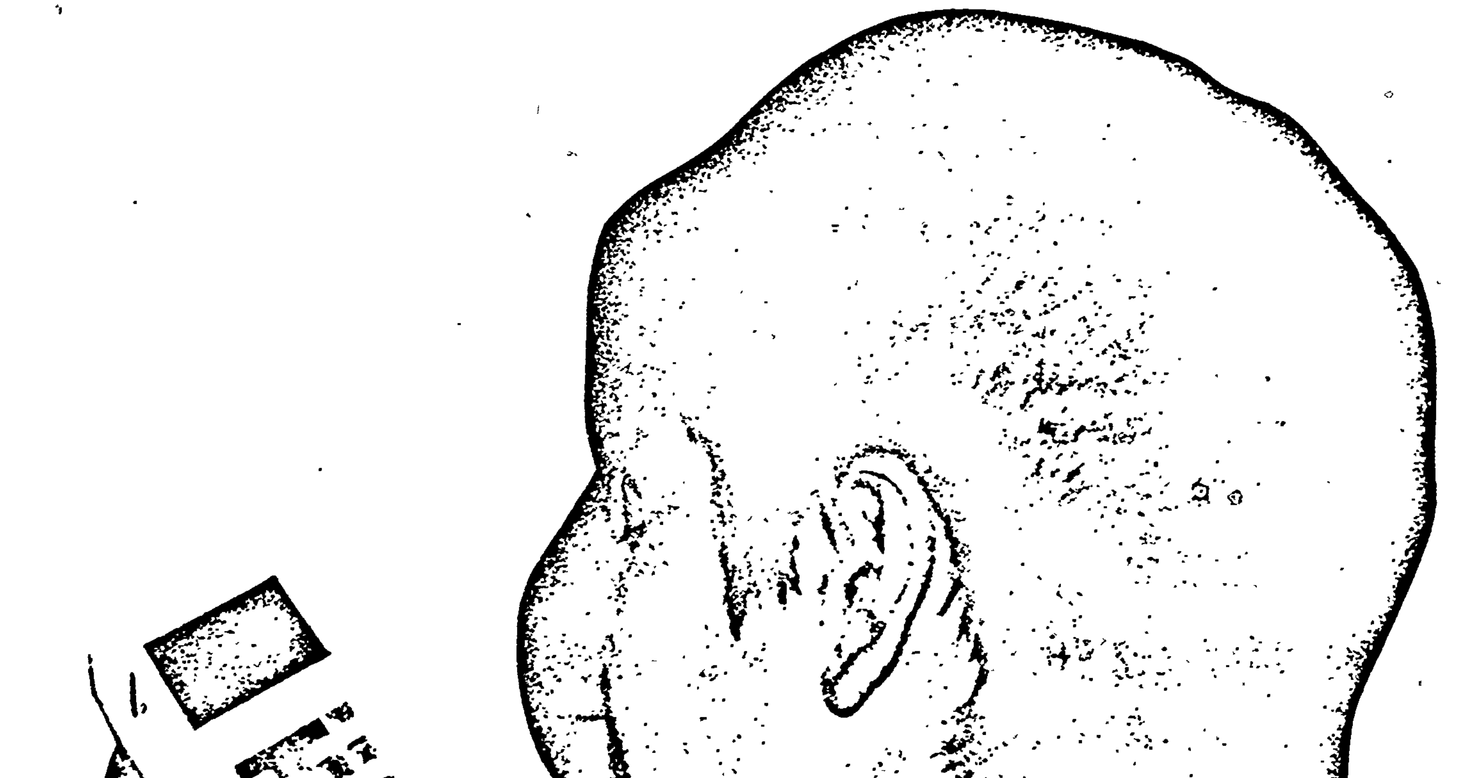
Problems which the vast majority of the class had been unable to solve without the calculator now became manageable for all. It was possible to concentrate on analyzing how to solve the problem once the calculator relieved the children of the cumbersome computation involved in arriving at the solution.

Motivation

Without exception the children found the calculator added excitement to math lessons. The fact that the weekly calculator lesson was a class activity, rather than the customary group or individual instruction, provided an interesting change. Children with little prior success in math were elated to find that they could also attain success. The more advanced pupils found the material challenging. They were encouraged to assist any classmate having difficulty. The children discovered that the calculator did not lend itself to every type of problem, but was invaluable in the more complex mathematical operations at every grade level. The general consensus was that the calculator is "fun." □

When You Use a Calculator You Have to Think!

By Phillis I. Meyer



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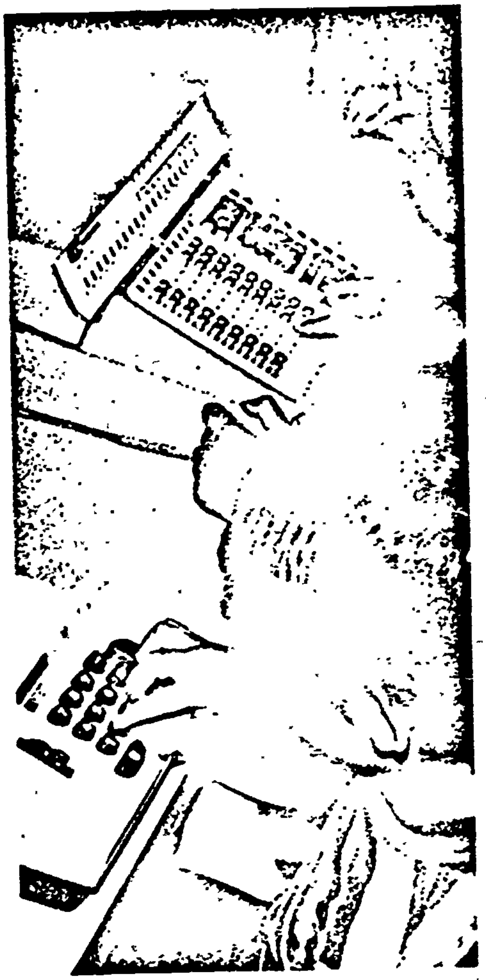

We had just finished a semester of using simple handheld calculators in our fourth-grade mathematics class. I had given every student a sheet of paper and asked each to write what, for them, was the hardest thing about using the calculator. I got a variety of answers—pushing the wrong key and forgetting to "clear" it, turning the calculator off before a problem was finished, making a mistake and having to start all over again, and most revealing of all, "When you use a calculator you have to *think*."

Several years ago we had secured several old office calculators, the noisy "clunkety-clunk" variety of the forties and fifties. The kids had had a great time adding and subtracting. But multiplying and dividing were more difficult since you had to wait for all those "clunkety-clunks." The paper-tape adding machine was better, but still multiplication and division were nearly impossible for my fourth graders.

For several years the sixth-grade mathematics students of our school have operated a School Store that carries school supplies such as pencils, paper, and so on, at a very low price. Profits were accumulating and we decided to purchase twenty-five simple four-function, rechargeable, hand-held calculators. We had earned half of the funds through the School Store and the generous PTA donated the other half. Having read many articles about the advantages and disadvantages of hand-held calculators in the classroom—in particular, the November 1976 issue of the *Arithmetic Teacher*, a special edition on minicalculators—I was ready to give it a try. My students were certainly eager to have "instant answers" for their arithmetic problems.

I began by using a series of activities designed to help students learn the function of each key. The + key was easy, so was the × key; but the tricky − key sometimes gave you a "−" in front of your answer. We explored to find why this happened and discovered that zero was *not* the beginning of the number line—there were negative numbers. We worked with negative numbers for a few days, trying, in particular, to understand when and why they might occur in everyday life—the thermometer, the family checkbook, and your allowance when you borrow on next week's allowance. We experimented with using the function keys with negative numbers.

Some students discovered that pushing a function key more than once would cause the calculator to keep repeating the process with the number last entered. We used this with the × key to talk about scientific notation and powers of numbers. The ÷ key gave us some very long, strange looking answers, usually with a decimal point somewhere in the string of numbers. Place value had to be rediscovered. The ones column was *not* always the column on the far right. Once the basic understanding of place value was developed, students could easily convert fractions to their decimal equivalents and add, subtract, multiply, and divide them.

Now that we knew what the function keys would do, we could get started. We played some simple games, with the children playing with partners. We took the School Store price sheet and "spent" all kinds of money. We bought things from catalogs and "played store," using the calculators. We discovered that some numbers looked like letters when we turned the display up-

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side down. We worked problems that gave us answers that would spell words if we got them correct. Students wrote their own problems to give "word" answers. Some of the problems became quite involved, with several types of operations required to get the answer. All of these activities acquainted the students with the capabilities of their calculators, as well as with some of the hazards involved in using them.

So far, all was a bed of roses. You punched certain keys and got certain answers. The game sheets all had "+", "−", "×", or "+" telling you what to do next. The buying activities were a little harder, but adding and subtracting sums of money was something the children were all familiar with, besides being lots of fun.

Probably the most valuable part of our project in using calculators was in solving some "real" problems. Now we had to *think*. We talked about how we would go about solving each problem. We used role playing on some of the harder ones. Did we want the answer to be a "big" number or a "small"

number? We began with some simple problems, solving them together as a class. After we had discussed the problem, the students worked it individually on their calculators. Then we asked ourselves, "Does our answer make sense? Is it about the right size?" If not, then we tried again. We learned to round numbers and to estimate. And we discovered that we still needed to know some basic arithmetic facts to do this.

We used our developing problem-solving skills in consumer mathematics situations with a variety of materials, some even on the junior high level. Sometimes the solutions got very involved. Our simple calculators had no memory and sometimes we had to use our paper and pencil to "save" numbers we needed to use later.

I made up problems with outrageous numbers to be computed. Students began to create their own story problems, using a variety of operations required for solution. Surprisingly, some of the most successful problem solving was done by the average students. The students who could "reason" through a problem and know what should be done were successful because they were not burdened or hampered by a lack of computational ability. The calculator could do that for them. The frustrated students were the ones who had always been told "what to do." Even though they could recite all the multiplication facts in two minutes, their reasoning abilities had not been developed. Soon, though, with much encouragement, these students began to experiment: Will I get a reasonable answer if I do this? No? Then I'll try this. They began to reason and to think, but not always before they acted. If they did have to "try again," it was not a defeating, laborious process because the calculator would do the calculations for them. They began to learn how to "reason through" a problem.

A change in attitude was evident very early in the project. Students who had previously been of the "I hate math" variety were now saying, "Do we have to stop now?" We still had some basic drill and practice, but the drudgery was gone when you could check the answer with a calculator and know right away if you were right or wrong.

As I look back over my lesson plans for fourth-grade mathematics, I wonder what some stranger would say upon seeing the results and outcomes of the fourth-grade mathematics lessons. Students had had experiences in—

- adding, subtracting, multiplying, and dividing with negative numbers;
- using powers numbers in correct scientific notation;
- place value, including 8-place decimals;
- converting any fraction to its decimal equivalent;
- adding, subtracting, multiplying, and dividing with decimals;
- reasoning logically to solve complicated story problems using two or more operations with whole numbers, fractions, or a combination of both.

Without the calculator, these processes would not have been possible for an average nine- or ten-year-old student.

The calculator is here to stay, at prices so low that nearly every child has access to some form of simple calculator. Through guided activities in the classroom, it can be a very valuable teaching tool in exploring a variety of mathematical topics. We used the calculator in numerous ways and learned many things. The most valuable benefit to most students was learning how to reason logically through a problem to reach a solution. We found that when you use a calculator, you have to *think*.

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Calculators for Kids Who Can't Calculate*

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It is not uncommon these days to find classroom sets of calculators available for use in mathematics instruction. ** However, when mathematics teachers at the high school level are asked what they do with them, a common response is that they're used with the Trig class or other advanced mathematics classes. These students find calculation tedious and a waste of time, and they certainly should be using calculators. (So do I and so I do!) For the lower level mathematics students, though, some teachers believe that calculators have no legitimate or appropriate use. If lessons for these students consist of worksheets devoted to computation practice, then this is true. Calculators turn such "lessons" into trivial and illegitimate exercises. On the other hand, there are many instances where calculators for these kids (who can't compute efficiently and accurately, anyway) turn the lessons into stimulating mental challenges. The following collection of possible uses of the calculator provides some examples to try!

Calculator Competition

One type of activity that can be used with almost any mathematics topic you're teaching at this level involves competitions with the calculator. Students without calculators are encouraged to compute arithmetical expressions mentally more quickly than students who are using the calculator. The choice of suitable expressions is crucial to the success of this activity. For example, you wouldn't use 497×368- - -. The calculator would win hands down, but you can use 400×700. Students who have had a brief review of multiplication of multiples of ten and who know that $4 \times 7 = 28$ can determine that 400×700 is the same as $28 \times 100 \times 100$ or 280,000 quicker than another student who is depressing $\boxed{4}$, $\boxed{0}$, $\boxed{0}$, $\boxed{\times}$, $\boxed{7}$, $\boxed{0}$, $\boxed{0}$, and $\boxed{=}$ on the calculator.

The psychological advantages of this type of activity include immediate positive reinforcement. There is a high probability of success for the students and their confidence will be increased. The competition is nonthreatening because they're playing against a machine and you'll rig it so they'll win!

* Paper Presented at the Annual Convention of School Science and Mathematics Association, Pittsburgh, Pennsylvania, November, 1977.

** This paper is a presentation of some ideas and activities that can be used with students in general mathematics classes at the high school level. Minor modification will make them appropriate for middle school or junior high school students.

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Management of this type of activity will depend upon the number of calculators you have available. With ten, or two, or even one calculator you can pit the Calculator-Kid(s) against the Thinkers. Cards, regular notebook paper or transparencies with expressions written on them should be prepared before class.

Expressions such as these can be used to review place value ideas:

$\boxed{7000 + 400 + 20 + 6}$ $\boxed{4 \times 100 + 3 \times 1}$ $\boxed{45800 \div 100}$ $\boxed{10^6}$

You may want the students to conclude on the last card that 10^6 is all right the way it is (as long as we're sure it's not 60 and is the same as $10 \times 10 \times 10 \times 10 \times 10 \times 10$ or 1,000,000).

Other expressions that involve computations with whole numbers are:

$\boxed{40 \times 60}$ Using basic facts of multiplication and the associative and commutative properties of multiplication, students can quickly determine that

$40 \times 60 = (4 \times 10) \times (6 \times 10) = (4 \times 6) \times (10 \times 10)$ or 24×100 or 2400.

$\boxed{4893 \times 631 \times 0}$ Recalling that for any whole number n, $0 \times n = 0$ makes this a quick mental computation.

$\boxed{0 + 97642}$ Remembering that for any non-zero whole number n, $0 - n = 0$ enables students to compute this with ease.

$\boxed{997 + 185}$ Using the idea that 185 can be renamed as $3 + 182$ helps translate this into an easier computation:

$$\begin{aligned} 997 + 185 &= 997 + (3 + 182) \\ &= (997 + 3) + 182 \\ &= 1000 + 182 \\ &= 1182. \end{aligned}$$

$\boxed{1234 - 999}$ Using the principle that for any whole numbers a, b, and c

$$a - b = (a + c) - (b + c)$$

yields $(1234 + 1) - (999 + 1)$ or $1235 - 1000$, and an easier computation to carry out.

$\boxed{5 \times 2468}$ Renaming 2468 as 2×1234 gives

$$\begin{aligned} 5 \times 2468 &= 5 \times (2 \times 1234) \\ &= (5 \times 2) \times 1234 \\ &= 10 \times 1234, \end{aligned}$$

a very easy computation.

Calculations Too Big for the Calculator

Using the most inexpensive calculators that can perform computations in addition, subtraction, multiplication and division but do not have scientific notation capabilites provides an opportunity to explore another method for performing computation of large products. This method makes use of the distributive property of multiplication over addition. That is, for any whole numbers a, b, and c, $a \times (b + c) = (a \times b) + (a \times c)$.

To establish the need for this approach to a particular computation students should try to compute an expression such as $19 \times 5{,}876{,}213$ with the calculator. In some way the calculator will indicate that this product is too large to display. (Mine flashes on and off.) A quick estimation discussion will reveal that the product is a little less than 120 million and more than 100 million. Then you're ready to perform the computation some other way:

$19 \times 5{,}876{,}213$	$= 19 \times (5{,}876{,}000 + 213)$	renaming the larger number as a convenient sum
	$= (19 \times 5{,}876{,}000) + (19 \times 213)$	using the distributive property of multiplication over addition
	$= (19 \times 5{,}876 \times 1000) + (19 \times 213)$	factoring so that there is a power of ten in the first expression (Powers of ten are "convenient" in this method.)

Now using the calculator to compute $19 \times 5{,}876$ and 19×213 we have

$$\begin{aligned}(19 \times 5{,}876 \times 1000) + (19 \times 213) &= (111{,}644 \times 1000) + 4047\\ &= 111{,}644{,}000 + 4047\\ &= 111{,}648{,}047\end{aligned}$$

Students can make up their own computations, estimate the resu' s, carry them out, and then exchange with other students as a check on th performance. They will be using some important mathematical ideas and the calculator will be doing the "hard" part.

Square Roots, Cube Roots, etc.

The inexpensive four-function calculator can be used to find accurate approximations for square roots, cube roots, fourth roots and so on. The ideas are the same. The realistic interpretation of roots becomes apparent with simple questions such as:

$\sqrt{46}$: What number multiplied by itself gives 46?
$\sqrt[3]{58}$: What number used as a factor three times gives 58?
$\sqrt[4]{85}$: What number used as a factor four times gives 85?

Estimation that uses multiplication facts that are relatively simple should be an integral part of exercises such as these. For example

$6^2 = 36$ and $7^2 = 49$ so $6 < \sqrt{46} < 7$

or

$3^3 = 27$ and $4^3 = 64$ so $3 < \sqrt[3]{58} < 4$

At this point attention must be given to place value ideas related to order and decimal representation of rational numbers. Selecting numbers between 6 and 7, or 6.7 and 6.8 or 6.78 and 6.79 involves important skills and concepts that deserve the review and extension that this type of activity provides. The following procedure is an example of classroom technique appropriate to determining a rational approximation for $\sqrt[3]{1048}$:

Consider $\sqrt[3]{1048}$		$10^3 = 1000$ (without a calculator)	$11^3 = 1331$ (with a calculator)
So	$10 < \sqrt[3]{1048} < 11$		
Try	10.1^3 ...... with a calculator .... 1030.301 (Not big enough)		
	10.2^3 ...... with a calculator .... 1061.208 (Too big)		
So	$10.1 < \sqrt[3]{1048} < 10.2$		
Try	10.15^3...... 1045.6783 in between		
	10.16^3...... 1048.772		
So	$10.15 < \sqrt[3]{1048} < 10.16$		
Try	10.158^3..... 1048.1528 in between		
	10.157^3..... 1047.8432		

So, correct to the nearest hundredth we can approximate the cube root of 1048 by 10.16, which is probably close enough for practical purposes. In the meantime sound mathematical principles have been reinforced and the computation is a breeze.

Prime Factorization

Loren Henry in the November, 1977 issue of *School Science and Mathematics* pointed out that calculators can be used efficiently to determine prime factorization for large (previously unmanageable) numbers. Again, review or reteaching of the prerequisite concepts is necessary, particularly in general mathematics classes. The definition of a prime number as being a whole number with exactly two whole number factors will determine the set of numbers from which to select trial divisors. This should be followed by a discussion of the fact that any whole number can be expressed as a product of prime numbers. Then students can participate in a sequence such as the following:

To find the prime factorization of 1066, we begin by considering the first prime number, 2. 1066 has 2 as a factor since the last digit of the numeral is "6".

$$1066 \div 2 = 533, \text{ so } 1066 = 2 \times 533$$

Now we proceed to find prime factors of 533. Since "533" does not end in either "0," "2," "4," "6" or "8", 533 does not have 2 as a factor and a calculator isn't necessary to determine this.

Primes		
2	$533 \div 2$	2 is not a factor (not necessary to compute)
3	$533 \div 3 \doteq 177.66666$	3 is not a factor
5	$533 \div 5$	5 is not a factor (not necessary to compute)
7	$533 \div 7 \doteq 76.142857$	7 is not a factor
11	$533 \div 11 \doteq 48.454545$	11 is not a factor
13	$533 \div 13 = 41$	13 is a factor

So $533 = 13 \times 41$. The other factor that was found is a prime number, as well, and we're finished. That is $1066 = 2 \times 13 \times 41$.

It is beneficial to point out to students that as the primes we divided by became larger the results of the computation became smaller. This can be used to determine where to stop in this procedure, and a quick glance at the calculator reveals that to us. When the number that we've divided by is larger than the result of our division, we've passed the square root of the number. Any factors that could be detected now would have been detected earlier by smaller primes, just by looking at the calculator. Suppose we're considering 521.

$$
\begin{aligned}
521 \div 3 &\doteq 173.666666\\
521 \div 7 &\doteq 74.428571\\
521 \div 11 &\doteq 47.363636\\
521 \div 13 &\doteq 40.076923\\
521 \div 17 &\doteq 30.647058\\
521 \div 19 &\doteq 27.421052\\
521 \div 23 &\doteq 22.652173 \qquad \text{We can stop here because}\\
23 &> 22.652173.
\end{aligned}
$$

This procedure works well with some numbers, those whose factors aren't obvious. However, students should realize that the procedure isn't appropriate for a number such as 3,000,000.

$3{,}000{,}000 = 3 \times 1{,}000{,}000$ by inspection.
1,000,000 has 10 as a factor six times.
Each of the six 10's has 2 and 5 as factors.

$$
\begin{aligned}
\text{So}\quad 3{,}000{,}000 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5\\
&= 2^6 \times 3 \times 5^6
\end{aligned}
$$

. . . without a calculator at all!

Patterns

Many interesting number patterns can be discussed with general mathematics students. If they are allowed to peform the calculations with a calculator there is more time to be spent on forming generalizations and testing hypotheses. The following are examples of some patterns for students to examine.

1. Products involving two 2-digit numerals where the numbers can be expressed as $10a + b$ and $10a + (10 - b)$ where a and b are natural numbers less than 10. The tens digits are the same and the sum of the ones digits is 10.

   For example, consider $76 \times 74 = 5624$.

   $$
   \begin{array}{ccc}
   \text{The tens digit is 7} & & \overbrace{7 \times (7+1)}\\
   76 \times 74 & = & 56\ 24\\
   \text{Sum is 10} & & \underbrace{6 \times 4}
   \end{array}
   $$

   Similarly, $48 \times 42 = 2016$ and $65 \times 65 = 4225$.
   A little algebra reveals that

   $$(10a + b) \times [10a + (10 - b)] = 100a(a + 1) + b(10 - b)$$

   $$
   \begin{aligned}
   \text{Thus } 65 \times 65 &= (100 \times 6 \times 7) + (5 \times 5),\\
   48 \times 42 &= (100 \times 4 \times 5) + (8 \times 2), \text{ and}\\
   76 \times 74 &= (100 \times 7 \times 8) + (6 \times 4)
   \end{aligned}
   $$

   The pattern can be used without being verbalized and it can be easily discovered by students at this level. The calculator can be used to check the examples and to provide counter-examples for faulty generalizations.

2. **Patterns** that encourage students to guess what comes next are fun to use in the classroom. Some students will be able to provide a reasonable mathematical analysis or justification. Others may not know why a pattern works, but, nonetheless, will be amazed that it does.

For example, consider the following patterns and predict what comes next in each:

$$6 \times 6 = 36 \qquad 56^2 - 45^2 = 1111$$
$$66 \times 66 = 4356 \qquad 556^2 - 445^2 = 111111$$
$$666 \times 666 = 443556 \qquad 5556^2 - 4445^2 = 11111111$$
$$12345679 \times 9 = 111111111$$
$$12345679 \times 18 = 222222222$$
$$12345679 \times 27 = 333333333$$

The preceding ideas are just a few examples of appropriate activities for students who are often unmotivated and even unable to perform in a satisfactory way. The calculator provides them with the opportunity to explore mathematical ideas . . . successfully. Let's use calculators with kids who *really* need them!

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Using Calculators with Juniors

BRIAN DAVIES
I.L.E.A.

It all started with the class teacher saying she would like to try something new! I had just been to a workshop on the use of pocket calculators in the classroom—so we had a deal; although neither of us realised how stimulating it would turn out to be.

The first stage was to obtain the calculators. The school only had two, so she asked the children. All the children who had them at home brought them in well in advance. One or two even had them bought specially, and I was pleased to see a dozen *working* calculators. The teacher had even gone to the trouble of getting spare batteries—an indication to the children of her interest and concern. So we had one between two children—just enough for the whole class to be involved in some things, and enough for one each in group work.

I thought we might start with a trick or two just to demonstrate the 'number crunching' power of the calculator.

Choose any three digit number, e.g. 365;
repeat it, i.e. 365 365;
divide by 13;
divide by 11;
divide by 7;
and see if the answer is the number you started with. (This also acted as a check to make sure they were working properly.)

The children were interested and worked on their own for some time, trying different numbers. This gave us the opportunity to talk to individual groups.

Could it be done with 2 digit numbers (put 0 in the middle)? single digit numbers? 4 digit numbers? (No!)

Next we started introducing games in groups of about four. Although we had copies of instructions for the games, by far the most effective method was to introduce the game to a group by playing it with them. The first one we used, and still one of the most popular, was simply a "guess the answer" game, played in groups of about four. One child makes up a sum of any sort. e.g.

346 + 48 + 274
or 36 × 18,
or half 3178,
or double 468, etc.

That child then writes it down on paper in the middle of the table and the other children write down their guesses of the answer on the same bit of paper. The child who invented the sum then works it out on the calculator and whoever was the nearest gets four points, the next three, and so on. It is then the turn of the next person in the group.

Only experience can tell you the best way to play this game, but we found it best if the questions were not made too hard, and if the group stuck to one sort of operation at each session. Halving and doubling big numbers went down particularly well, giving an opportunity for talk about large numbers.

Another popular game was "Zero's the limit", played by two children with one calculator.

First player enters a two digit number;
second player punches minus, then a single digit number, then '=';
each player punches minus, then single digit number, then '=';
but each number must be vertically, or horizontally, adjacent to the previous number pressed.
The '0' button may not be used.
The player who makes a minus sign appear on the display is the loser.

The sort of games where you aim for a 'goal' are very adaptable for use with a calculator. With one group of four the goal was 29. We all started with zero and could add any single digit number in turn. If you were the one who ended up making 29, then you were the loser This game was very popular and gradually the rules became more complicated, highlighting knowledge of tables!

If you landed on an even number you lost a life;
if you landed on an odd number you lost a life;
if you landed on the three times tables you lost a life;
if you landed on the four times table you lost a life;
if you landed on the three or *four times table you lost a life;*

and so on, whoever lost the least number of lives being the winner.

Although puzzles and games were popular and greatly increased the children's awareness of number, we both felt that if calculators were to catch on and hold a permanent place in the classroom they should be of benefit for more than just puzzles and games.

Could they help with the children's normal

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mathematics—decimals, tables, place value, four rules?

Could they help with work from their maths schemes?

Could they be used to enable children to tackle *real* situations which would not have been feasible without calculators, i.e. could they enlarge and enrich the curriculum by

a) highlighting which arithmetic operation should be used in various circumstances,

b) allowing the children to tackle situations where the numbers and calculations involved would otherwise have been out of their grasp,

c) throwing up answers which are not always understood?

We decided to tackle the last first, wondering how much of the previous would be used. We need not have wondered so much since, once the children saw a *need*, they made great strides in their understanding of such things as large numbers, place value, decimals and the four rules.

So, what situations could we see, within the children's understanding, which would benefit from the use of calculators? We sat and looked around and talked to the children. Alas, nothing came up, until I walked past the gerbil asleep in his cage and noticed for the first time how fast he breathed. So this became our starting point, although in retrospect I think there could have been any number of them. This started us on a series of question and problems, some of them quite eccentric, but none the less still holding a lot of interest for the children.

How many times does the gerbil breathe in a day?
How many times does the gerbil breathe in a year?
How many times do we breathe in a day/year/lifetime?
How many times does our heart beat in a day/year/lifetime?
How many seconds in a day?

With the help of calculators these problems came within the children's grasp, although none found them particularly easy.

For example, when they tried "How many seconds in a day?" it highlighted first of all that not all the children (fourth years) knew how many seconds in a minute, minutes in an hour, or hours in a day—things that the teacher assumed they knew! Then because nobody wrote down this information, part of it was consistently omitted in the calculations and many did $60 \times 24 = 1440$ seconds in a day! Other children did not use the multiplication function and just wanted to keep on adding 60. They, of course, lost count of how many 60s they had added.

Yet another child did $60 + 60 = 120$ seconds in an hour, then $120 \times 24 = 2880$ seconds in a day. Another did $60 + 60 = 120$, then added 120 twelve times on the calculator to get 1440, then added another 1440 to get 2880 seconds in a day.

Peter tried:

60	seconds in 1 minute
120	seconds in 2 minutes
180	seconds in 3 minutes
240	seconds in 4 minutes
300	seconds in 5 minutes
600	seconds in 10 minutes
1,200	seconds in 20 minutes
3,600	seconds in 60 minutes
7,200	seconds in 2 hours
14,400	seconds in 4 hours
28,800	seconds in 8 hours
57,600	seconds in 16 hours
86,400	seconds in 24 hours

These sorts of problems highlighted three particular skills:

a) solving a big problem by breaking it down into manageable steps;

b) selecting the appropriate operation on the numbers;

c) interpreting your answer—

e.g. How many seconds in a day?
Does 60×24 give us seconds in a day, seconds in an hour, or hours in a day?

The sort of questions mentioned above seem to hold their own intrinsic interest although, of course, they would not be suitable for all ages. I have come across other, perhaps even more weird problems that have been tackled.

How many hairs on your head?
How many names in the telephone directory?
How many blades of grass in a field?
How many wooden blocks in the hall floor?
How many words in a book?
How much does it cost to light the school each year?
If 45,000 attend a football match and there are nine sections, how many people in each section? If there are 50 rows in a section, how many in a row? One policeman is needed for each 400 people, how many policemen needed for each section? How many for the whole ground? At £15.00 per policeman, how much will it cost for the afternoon? How much is that on each ticket sold?

This last story was tried because the teacher was concerned about how the children tackled division problems, and in fact her concern was most justified, as illustrated by a single problem.

Share 48 sweets between 16 children.

Half of the class of 11-year-olds tried $48 \div 16 = 3$ on the calculators, whilst the rest chose to do $16 \div 48 \doteq 0{\cdot}3333$ . . . . Why? Because they really did not understand the nature of division, or because they are used to this notation:

Whatever the reason, the children seemed a lot happier about operating division after further problems of this nature and stories like the above. (At one stage we actually had to get the Unifix out to prove who was right. 48 Unifix shared out to 16 children gave them three sweets each. Half the calculators showed not three but 0·3333 . . . . Some children were so sure they had done the operation the right way round that they accused the calculators of not working properly!)

Two other areas of mathematics crop up very soon after children start using calculators. They very soon come across decimal points on their displays and, equally as soon, come across large numbers. Because of their involvement with their investigation they can be motivated and there exists an ideal teaching opportunity. Children's reactions to strings of decimals are interesting, varying from "The calculator has gone wrong" to ignoring the decimal numbers altogether. In fact it seems that children and adults can operate happily with decimals without a real understanding of what they are.

When decimals come up for the first time with Derek he asked what it was. I said, "It is a tenth. Do you know what that is?"

"No," he replied. "Is it something like a quarter?"

Eventually he decided to ignore the decimals completely in his calculations, although when he pronounced his eventual answers he said, "Except it's a bit more than that really!"

This is an important aspect of the use of calculators in this sort of investigation. We found that children already knew mathematics we had *not* taught them, and also did *not* know things that we thought they might or even should know.

One boy had wildly suggested, "How many blades of grass on the field?" The teacher, never one to miss a likely opportunity, said O.K. This boy seemed to appreciate straight away the need for sampling or grouping, i.e. he realised he could not count all the grass and he thought quite quickly of using a small square to start with. Many children would not see this for themselves and it is a key stage in investigations of this sort. He made his sample in a square centimetre and then did a second, only to find he had different answers. His teacher suggested trying a few more. He did 5 and then added them up and wanted to *halve* the answer to get a good number. However, when he did this he got a number far higher than any of his original samples. His teacher suggested dividing by five. He did this and came up with a decimal he could not understand. So two sources of work had arisen—averages and decimals—which could be followed up in more depth at a later stage.

He eventually decided to ignore the decimal fraction—not a bad decision considering that the whole exercise was one of approximation—and proceeded to a square metre. To his amazement the teacher now found out that this boy, normally 'good at maths', did not know how many centimetres there were in a metre, let alone how many square centimetres in a square metre—so out with the calculator and paper. Next came measuring the field, starting with a trial patch "to see if it would really work". It did, so he measured most of the field but left out the awkward bits—another follow-up for the teacher. Eventually came the first calculation involving multiplying by 10s, 100s, 1000s and he ran out of space on the display panel. So he and the teacher had a long talk about what happens to numbers when you multiply by 10, 100, or 1000 and eventually the child did the calculation without the zeros.

At the end of this, she knew further work was needed on decimals (when is it not needed!), large numbers, averages and area, but at least the child had begun to see the need for such ideas. Furthermore he is now starting to compare fields and lawns, under trees and in the sun, gaining some insight into what calculations are needed in real circumstances.

After the class had been working on these things for about a month we wondered where to go next. I tried to encourage the teacher to try the calculator within the mathematics scheme. However, she was still unhappy about the children's *use* of the four rules of number. She was still doubtful as to whether they could apply them in real situations—a problem highlighted in the recent APU report on 11-year-olds. Could children actually construct the appropriate mathematical model to help them solve real problems?

We once again relied on the children for ideas and this time we asked them to think of a 'how many' or 'how much' type question that would give them large numbers in the answer, and that involved the school or the people in it. This time there was more response and the children suggested problems that ranged from th quite simple to the very complicated. Some could be counted quite easily; others needed extensive use of the calculators. All of them involved breaking the problem up into manageable parts and operating on these parts.

How much money did Alan, who bought a packet of crisps and a bottle of Tizer each day, spend in a week?
How much in four years in the Junior school?
What could he have bought instead—a motorbike?
How much milk gets drunk each day/year in the school?
How many cows would the school need to keep?
How many exercise books in the school?

One girl later branched out into

How many pages?

How many squares?

How much did they weigh?

How much did one square weigh?

How many squares made a gram?

How many bricks in a wall?

Later on a boy whose father is a bricklayer went on to:

How long to make the wall?

How many journeys from the pile of bricks to the wall for the hod-carrier?

How many lorry loads?

How many bricks in a cubic metre?

How many doors in the school?

Then later:

How many hinges?

How many screws?

How much do the children in the school weigh?

Do they weigh the same as a double decker bus?

An elephant?

Does it give false confidence to some? Timothy played the guessing game by himself, with a chart like this.

NUMBER	OPERATION	GUESS	CALCULATOR CHECK
43	+1 +10 +100 +1000 −1 −10 ×1 ×10 ×100 +50 +150 +250 etc . . .		

He was not guessing but simply working it out on the calculator straight away. This seems to be where the teacher has to make a judgement, as she has to about many activities in the classroom, as to whether the calculator and/or the activity is suitable for that particular child. It might well be that a group game, played with children of his own ability, could stop him 'cheating' and still increase his confidence.

Thoughts after two months work

How exciting and interesting it has all been.

How it has converted the teacher to an investigational approach to mathematics.

What a liberating force the calculators have been, enabling the children to tackle calculations that would have been tedious, time-consuming and error-prone without them.

How important is the teacher's attitude, giving the children time, taking their ideas seriously, taking time for discussion and being interested in the *process* as well as the answer.

The importance of discussion about sampling, and grouping, writing down the calculations beforehand if possible, so avoiding the situation of

"How did you get that answer?"

"Er, don't know, forgotten, er . . . think it was . . ."

i.e., planning the investigation and breaking it into manageable chunks.

How it enabled the children to think of a couple of problems *and* gave them opportunities to solve them.

How the less able children developed an enthusiasm, especially with the games.

How, by watching the children use the calculator, a teacher can *check* how sure a child is with the operational side of the work.

How hesitatingly are the buttons pressed?

Are errors made with × and ÷?

Are they fluent with the ÷ symbol?

How this is leading onto other things in that the children are asking about the buttons which are not being used (i.e. memory, $\sqrt{\ }$, x^2, σ_0).

In fact it seems important to have calculators permanently available as an essential tool to be used as occasion arises in classroom discussion and work.

Calculator Use In The Middle Grades

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RATIONALE

The world is presently experiencing an explosion of electronic technology which is threatening to render obsolete much of what we teach in school mathematics. Electronic calculators, unknown in the early 70's, are so prevalent that they can be found in nearly every home or office. Because of their low cost and widespread use in society, calculators have the potential of reshaping the mathematics curriculum in our schools. School practices generally lag far behind societal changes and the utilization of calculators in schools is no exception. While on-the-job and in-the-home use of calculators to perform complex computations is nearly standard, schools have yet to incorporate calculators into the mathematics and science curricula. This must change.

When asked to identify problems in schools today, teachers quickly list pupil apathy and lack of motivation as a major concern. Studies by Wheatley [1] and Wheatley et. al. [2] show that the calculator is a highly motivating instructional tool. Teachers report that pupils will tackle problems with the calculator they would never attempt otherwise. Activities presented later in this article provide examples of motivating mathematical material made possible through calculator use.

Often students are not successful in learning and applying concepts because the need to compute with paper and pencil may obscure the targeted mathematical ideas. However, with calculators to perform burdensome computations, the learner can focus on concepts, applications, and problem solving. The advantage is particularly evident for slow learners who are greatly aided in learning by a computational device. Likewise the more capable students can focus on problem solving heuristics and advanced topics. Teachers can present problems using realistic data (large numbers, decimals) when calculators are available to perform complex computations. These factors allow emphasis to shift from learning computational rules to applying mathematics in meaningful contexts. Thus we see that calculators in the classroom can be highly motivating and fa-

1. Wheatley, C. "Calculator Use and Problem Solving Performance." *Journal for Research in Mathematics Education*, in press.
2. Wheatley, G., Shumway, R., Coburn, T., Reys, R., Schoen, H., Wheatley, C., and White, A. "Calculators in Elementary Schools," *Arithmetic Teacher*, 27, (September 1979), pp. 18-21.

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cilitative of learning. As Phillips [3] says we must, ". . . nudge a student into organizing things in his head, putting the pieces together to build an ever-enlarging structured whole . . . ."

Described below are four activities which capitalize on the calculator as an instructional aid. A brief description of each activity is presented with suggestions for the teacher. These activities may be adapted to other grade levels or ability groups.

CALCULATOR ACTIVITIES AND SUGGESTIONS FOR IMPLEMENTATION IN THE CLASSROOM

Activity One: Estimating sums and addends
Purpose: To develop estimation skills and order relations
Grade Level: May be adapted to any grade
Procedure: 1. Present the problem using the diagram below.

$137 + \_\_\_ = \_\_\_\_\_$

If the ball must land in the range shown, what distance must the ball be kicked? What number added to 137 will put you between 450 and 470?

2. Write suggested answers on the board.
3. Encourage pupils to explore all possibilities.
4. List all answers.
5. Ask, "How many answers are possible? Why?"

Extension activity (multiplication and division of whole numbers)

1. Use the representation shown.
   Present problem $56 + \_\_ = \_\_$
   Find a whole number which multiplied by 56 puts you between 750 and 900.
2. Write suggested answers on the board.
3. Obtain agreement on the solution set.
4. Ask, "Why are there only three numbers in the solution set?"
5. Supply students with similar problems to solve.

3. Phillips, J. "Go Back to the Beginning? Where's That?" *School Science and Mathematics*, LXXX (February 1980), p. 138.

Activity Two: Problem solving

Purpose: To develop problem solving skills, to reinforce the use of metric units, to illustrate the use of mathematics in health education.

Grade Level: Grades 5 to 8

Procedure:

1. Present the problem.
   Your heart usually pumps about 65 milliliters of blood per heart beat. If your heart beats 68 times a minute, how much blood is pumped in 1 minute? In 1 hour? In a 24-hour day? In 1 week?
2. May supply hints below:
   a. Could you make a chart?
   b. Try simplifying the problem, e.g., suppose the heart beats 2 times a minute.
   c. Is your answer reasonable?
3. When Paul is jogging, his heart beats 145 times per minute. How much blood does his heart pump in 1 minute? In 16 minutes?
4. Let students discuss their solutions.
5. Present a different problem such as:
   Sarah's heart pumped about 4.7 liters of blood in 1 minute. How many times was her heart beating?
6. May supply hints below:
   a. How is this problem like the problems above? How is it different?
   b. Try making a guess and checking it in the problem.
   c. Is your answer between 60 and 90 beats per minute?
7. Have students find their own pulse rate. Ask, "How much blood does your heart pump per minute?"

Activity Three: Application

Purpose: To make comparisons using division, to reinforce the use of metric units, and to develop the concept of rate.

Grade Level: Grades 6 to 8

Procedure:

1. Present the problem. Whose vehicle is the most energy efficient?
2. Discuss computation of kilometers per liter using the formula below:
   Kilometers ÷ Number of liters used = Kilometers per liter

3. Provide students with the chart below:

	Distance	Gas Used	Kilometers Per Liters
Mr. Remo	161 kilometers	11 liters	________
Ms. Kessler	435 kilometers	28 liters	________
Mr. Carter	289 kilometers	24 liters	________

4. Ask, "How does your family car compare?"
5. Challenge students to find the kilometers per liter ratings for the leading automobiles.

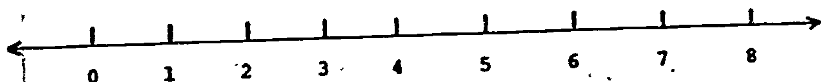
Activity Four: Developing the concept of decimal

Purpose: To develop the concept of .1, to compare decimals, to develop decimal ordinal sense.

Grade Level: Grades 5 to 8

Procedure:

1. Teacher states, "Let's teach the calculator to count. Press 1 + = = = ..."
   (Most calculators have a constant addend and will count but you should check the logic and keystroking of calculators being used.)
   "Count to 100 by ones."
2. "Now let's teach the calculator to count by tenths. Press .1 = = = ... Count to 10. How many times did you press = ?
   Does 3.6 come before or after 3.4?
   Does 5.1 come before or after 1.5?
   Which is smaller, 6.5 or 3.7?
   Where is 4.2 on the numberline?"



3. Have the pupils count by tenths again. Ask, "When counting by tenths, what comes after .9?
   2.0? 2.5? 2.9?"
4. Have pupils count back from 10 by tenths.
   10 − .1 = = = ...
5. Ask, "What comes before 2.0? 3.1? .9? 10?"
6. The concept of hundredths can be developed in a similar manner.

Many excellent calculator activities can be found in the sources below:

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SUMMARY

Calculators have the potential for reshaping our computationally oriented curriculum. Its place in the classroom is presently unknown. But it clearly has a *place*. The activities in this article suggest ways calculators can be effectively used to develop four types of mathematical thought.

CALCULATOR LESSONS INVOLVING POPULATION, INFLATION, AND ENERGY

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One of the joys of studying mathematics comes from the surprising results that are often obtained. An example of such a situation is the Rule of 72. This rule states that if money is invested at r percent compounded annually, the amount will double in approximately $72/r$ years. On the other hand, if the interest is compounded semiannually, the doubling time is approximately $70/r$ years; and if the interest is compounded instantaneously, then the doubling time is approximately $69.3/r$ years. In the interesting article "The Rule of 72," which appeared in the November 1966 issue of the *Mathematics Teacher*, Brown argues that these results are appropriate for "small" r. His argument, which makes use of logarithms, is not reproduced here.

The purposes of this article are (1) to show how a hand-held calculator can be used to help students discover the Rule of 72 and (2) to indicate how the Rule of 72 can be used to investigate problems involving population, inflation, and energy reserves. The lessons described in this paper are lessons that each author has used individually with both college and secondary students.

Lesson 1

The first lesson is introduced with the story about the king and the inventor of the chess game. The king wishes to reward the inventor, who is also a mathematician, and asks how he can do this. The mathematician-inventor asks for the wheat obtained by putting one kernel of wheat on the first square of the chessboard, two kernels on the second square, four on the third square, eight on the fourth square, and so on for the entire sixty-four squares of the board.

The observation is made that the total amount of wheat on the chessboard is

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{63}$$

grains, and

$$\begin{aligned} 1 + 2 &= 3 = 2^2 - 1, \\ 1 + 2 + 2^2 &= 7 = 2^3 - 1, \\ 1 + 2 + 2^2 + 2^3 &= 15 = 2^4 - 1, \\ 1 + 2 + 2^2 + 2^3 + 2^4 &= 31 = 2^5 - 1, \\ &\cdots \\ 1 + 2 + 2^2 + \cdots + 2^{n-1} &= 2^n - 1, \\ &\cdots \end{aligned}$$

and thus,

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{63} = 2^{64} - 1.$$

The statement is made that 2^{64} kernels of wheat is estimated to be approximately 500 times the total present yearly world wheat production (Bartlett 1976).

The purpose of introducing this story is to emphasize that in the case of a continuous doubling circumstance, the numbers get very large very quickly. Some students are interested in determining how large a number 2^{64} is. For those students having calculators with an x^2 function, the calculation of

$$2^{64} \doteq 1.8447 \times 10^{19}$$

is simple. Students are impressed by the rapid change in the display from 2^{16} to 2^{32} to 2^{64}.

Next, the formula for compound interest with interest compounded yearly is developed. This formula is

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$$A_n = p\left(1 + \frac{r}{100}\right)^n,$$

where A_n represents the total amount at the end of n years, p represents the principle, and r percent represents the rate. This result is derived in a typical textbook procedure.

Next, inflation is discussed. The point is made that a constant annual inflation rate is an application of compound interest. Students are asked what they think the cost of a new bicycle would be in the year 2146, assuming it now costs \$100 and that there will be a 6 percent annual inflation rate. (Most students predict less than \$500, whereas the actual amount is over \$1 500 000.)

No attempt is made to find the solution to the problem of the cost of the bicycle in the standard way, that is, an evaluation of $A_{165} = 100(1.06)^{165}$. Such an evaluation would involve the use of logarithms (which some students have not studied) or the use of the exponential function y^x (which some calculators do not have); but these approaches would detract from a strategy that is perhaps more important than the numerical result.

The preferred solution to the bicycle problem, and other similar problems, necessitates introducing the concept of doubling time, that is, the number of years it will take a certain amount of money to double in value when invested at a compound yearly interest rate of r percent. Students are easily convinced that the original amount of money is arbitrary; therefore \$1 is used. To find the doubling time, it is necessary to find n such that

$$A_n = 1\left(1 + \frac{r}{100}\right)^n = \left(1 + \frac{r}{100}\right)^n \doteq 2.$$

Students then calculate n's for several given possible r's. For example, if $r = 6$ percent, the following values are obtained with the calculator showing that $n = 12$ when $r = 6$ percent.

$$(1.06)^1 = 1.06$$
$$(1.06)^2 \doteq 1.12$$
$$(1.06)^3 \doteq 1.19$$
$$\ldots$$
$$(1.06)^{11} \doteq 1.90$$
$$(1.06)^{12} \doteq 2.01$$

These data are recorded in table 1. Circled numerals are determined by the use of the calculator and are integral approximations for n. Next, the students are requested to investigate this table. Some students will indicate that in each case the product of the percentage and the doubling time is 72. This conjecture is checked by the class. The question posed is How could we calculate the doubling time, given the rate? Students typically respond that the doubling time is $72/r$.

TABLE 1

r	Doubling Time in Years (Approximately)
3	24
4	18
6	12
8	9
9	8
12	6
18	4
24	3

Returning to the topic of inflation, students are asked to help complete table 2. The entries in the first two columns are given and all others are completed (circled numerals), assuming a 6 percent annual inflation rate. For this example, the doubling period is 12 years (72/6). The entries in the third column are double the corresponding entries in the second column, since one doubling period will have elapsed. The entries in the fourth and fifth columns are derived similarly. The date for the last column, 2146, is 132 years (11 doubling periods) after the date for the previous column. Hence, the entries in the last column are found by multiplying the corresponding entries in the previous column by 2^{11}.

The world population in 1975 was approximately 4 000 000 000, or 4×10^9, and the annual population increase has been fairly constant at about 2 percent (1976 Census, p. 866). Table 3 is then completed, assuming the rate of increase in population will remain at 2 percent. Observe that the

TABLE 2
Costs at a 6 Percent Annual Inflation Rate

Item	1978 Cost	1990 Cost	2002 Cost	2014 Cost	2146 Cost
Soft drink	25¢	50¢	$1	$2	$4096
Movie ticket	$3	$6	$12	$24	$49 152
Bicycle	$100	$200	$400	$800	$1 638 400
Compact automobile	$4000	$8000	$16 000	$32 000	$65 536 000
Modest house	$30 000	$60 000	$120 000	$240 000	$491 520 000

doubling time is 36 years. As before, the circled numerals are calculated by the class. Next, the observation is made that the total land area (including inland lakes and rivers) is approximately 1.36×10^{14} square meters (1976 Census, p. 867). Assuming a constant 2 percent population growth, there would be approximately one person for each square meter in the year 2515. A calculator is not needed to complete this table but it does make the computation easier.

A rather complete class development of this lesson takes approximately one hour. At times, only portions of the previous two tables are completed in class, and the students are asked to complete these tables as an out-of-class assignment. (In one instance this lesson was used alone, without the next lesson, when only one hour of class time was available.)

The first class is ended by presenting the students with the following problem:

> Many years ago a fictitious king had 2000 barrels of wine in his cellar. Given that his court consumed 20 barrels of wine last year, and past increases in consumption would indicate a 12% annual increase, how long will the king's wine supply last?

The students are asked to guess at a solution and then challenged to solve the problem by whatever means they can devise. Most students guess many years more than the actual answer of approximately 22 years. Sometimes students are able to solve the problem correctly prior to the next class

TABLE 3
World Population Assuming 2 Percent Growth Rate

Year	Predicted World Population
1975	4×10^9
2011	8×10^9
2047	1.6×10^{10}
2083	3.2×10^{10}
2119	6.4×10^{10}
2155	1.28×10^{11}
2191	2.56×10^{11}
2515	1.31×10^{14}

meeting but only after considerable effort. The difficulty they have solving the problem generates considerable interest in the methods that are developed in the second lesson.

Lesson 2

The second lesson is involved with the investigation of problems concerning how long energy reserves (coal and petroleum) would last, assuming a constant annual percentage increase in usage. These problems (e.g., the wine problem) are much more complex than the problem of determining how much energy reserves would be used in a given year (e.g., the inflation and population problems). In order to solve these more difficult problems easily, let x be the initial amount of fuel used, x_i the amount of fuel used in the ith year after the initial year, and $r = 6$. Next, the data in table 4 are presented. Of particular interest is the fact that

$$\sum_{i=13}^{24} x_i \doteq 2\sum_{i=1}^{12} x_i$$

and

$$\sum_{i=25}^{36} x_i \doteq 2\sum_{i=13}^{24} x_i \doteq 4\sum_{i=1}^{12} x_i .$$

These data are determined by use of a hand-held calculator. For example, $x_6 = (1.06)^6 x$. Many students recognize that the numbers 1.06, 1.1236, 1.1910, and so on are the same numbers that were obtained in the completion of the doubling-time table of the first lesson, using $r = 6$.

The observation is made that for $1 \leq i \leq 12$,

$$x_{i+24} \doteq 2x_{i+12} \doteq 4x_i .$$

This result is to be expected since the doubling period is 12 years when $r = 6$. Of particular significance is the fact that $\sum_{i=1}^{12} x_i$ is the amount of fuel used in the first doubling period, that $\sum_{i=13}^{24} x_i$ is the amount of fuel used in the second doubling period, and that $\sum_{i=25}^{36} x_i$ is the amount of fuel used in the third doubling period. Letting $\sum_{i=1}^{12} x_i = y$, the amount of fuel used in the second doubling period is approximately $2y$, the amount of fuel used in the third doubling period is approximately $4y$, the amount of fuel used in the fourth doubling period is approximately $8y$, and the amount of fuel used in the nth doubling period is approximately $2^{n-1}y$. These conclusions suggest that it is important to develop a technique for finding y without using the rather tedious computation presented above.

A point is made that $\sum_{i=1}^{12} x_i$ is the sum of the terms of a geometric progression and that it is easier to approximate this sum by using an appropriate arithmetic series. If $a_1 = x$, $a_{12} = 2x$, and $d = 1/11(x)$, then

TABLE 4

$x_1 = 1.06x$	$x_{13} \doteq 2.1329x \doteq 2x_1$	$x_{25} \doteq 4.2919x \doteq 2x_{13} \doteq 4x_1$
$x_2 \doteq 1.1236x$	$x_{14} \doteq 2.2609x \doteq 2x_2$	$x_{26} \doteq 4.5494x \doteq 2x_{14} \doteq 4x_2$
$x_3 \doteq 1.1910x$	$x_{15} \doteq 2.3966x \doteq 2x_3$	$x_{27} \doteq 4.8223x \doteq 2x_{15} \doteq 4x_3$
$x_4 \doteq 1.2625x$	$x_{16} \doteq 2.5404x \doteq 2x_4$	$x_{28} \doteq 5.1117x \doteq 2x_{16} \doteq 4x_4$
$x_5 \doteq 1.3382x$	$x_{17} \doteq 2.6928x \doteq 2x_5$	$x_{29} \doteq 5.4184x \doteq 2x_{17} \doteq 4x_5$
$x_6 \doteq 1.4185x$	$x_{18} \doteq 2.8543x \doteq 2x_6$	$x_{30} \doteq 5.7435x \doteq 2x_{18} \doteq 4x_6$
$x_7 \doteq 1.5036x$	$x_{19} \doteq 3.0256x \doteq 2x_7$	$x_{31} \doteq 6.0881x \doteq 2x_{19} \doteq 4x_7$
$x_8 \doteq 1.5938x$	$x_{20} \doteq 3.2071x \doteq 2x_8$	$x_{32} \doteq 6.4534x \doteq 2x_{20} \doteq 4x_8$
$x_9 \doteq 1.6895x$	$x_{21} \doteq 3.3996x \doteq 2x_9$	$x_{33} \doteq 6.8406x \doteq 2x_{21} \doteq 4x_9$
$x_{10} \doteq 1.7908x$	$x_{22} \doteq 3.6035x \doteq 2x_{10}$	$x_{34} \doteq 7.2510x \doteq 2x_{22} \doteq 4x_{10}$
$x_{11} \doteq 1.8983x$	$x_{23} \doteq 3.8197x \doteq 2x_{11}$	$x_{35} \doteq 7.6861x \doteq 2x_{23} \doteq 4x_{11}$
$x_{12} \doteq 2.0121x$	$x_{24} \doteq 4.0489x \doteq 2x_{12}$	$x_{36} \doteq 8.1473x \doteq 2x_{24} \doteq 4x_{12}$
$\sum_{i=1}^{12} x_i = 17.8819x$	$\sum_{i=13}^{24} x_i = 35.9832x$	$\sum_{i=25}^{36} x_i = 72.4042x$

$a_1 = 1.00x$
$a_2 = 1.\overline{09}x$
$a_3 = 1.\overline{18}x$
$a_4 = 1.\overline{27}x$
$a_5 = 1.\overline{36}x$
$a_6 = 1.\overline{45}x$
$a_7 = 1.\overline{54}x$
$a_8 = 1.\overline{63}x$
$a_9 = 1.\overline{72}x$
$a_{10} = 1.\overline{81}x$
$a_{11} = 1.\overline{90}x$
$a_{12} = 2.00x$

These values are compared with the corresponding values of x_1 through x_{12}.

$$\sum_{i=1}^{12} a_i = \frac{x+2x}{2}(12) = \frac{3}{2}x \cdot 12 = 18x.$$

Since $\sum_{i=1}^{12} x_i = 17.8819x$ and $\sum_{i=1}^{12} a_i = 18x$, using $\sum_{i=1}^{12} a_i$ in place of $\sum_{i=1}^{12} x_i$ results in an error of less than 1 percent,

$$\frac{.1181x}{17.8819x}.$$

The important generalization is made that $y \doteq 3/2(xm)$, where m is the length of the doubling period.

Purists are probably shocked that an arithmetic series is used to approximate a geometric series on the basis of a single example, but students are willing and able to accept this obvious impreciseness for the sake of convenience. Also, estimates of available energy reserves are questionable, at best, as a result of ecology constraints, prices of a given resource, and technological advancements. As long as the computed results are accepted for the rough estimates they are, the authors feel justified in using reasonable mathematical approximations.

The lesson is continued by

letting A be the amount of fuel available (estimated reserves) and

letting n be the number of doubling periods the fuel will last.

The contention is that

$$\begin{aligned} A &= y + 2y + 4y + 8y + \cdots + 2^{n-1}y \\ &= y(1 + 2 + 4 + 8 + \cdots + 2^{n-1}) \\ &= y(2^n - 1). \end{aligned}$$

$$\frac{A}{y} = 2^n - 1 \quad \text{and}$$

$$2^n = \frac{A}{y} + 1.$$

In 1977 the world petroleum reserves were estimated at about 6.46×10^{11} barrels, and about 2.17×10^{10} barrels were consumed that year (*Energy Statistics* 1978). Jointly, table 5 is completed (circled numerals) using the information given in the first three columns.

The only column in table 5 that is difficult to complete is the next-to-last column giving values of n for which $2^n = A/y + 1$. Some students have studied logarithms previously and are able to quickly recall that the solution is given by

$$n = \frac{\log\left(\frac{A}{y} + 1\right)}{\log 2}.$$

For those students unable or not wanting to use logarithms, the graph of $f(n) = 2^n$ (fig. 1) is used. To make sure that the students recall what the graph of $f(n) = 2^n$ looks like, a few of the points are plotted before placing a transparency of the graph on the overhead projector. From the value of $A/y + 1$ on the $f(n)$ axis, students are directed to go horizontally to the graph of the function, then down vertically to find the value of n such that $f(n) \doteq A/y + 1$. For example, if $A/y + 1 = 1.55$ then $n \doteq 0.63$. The values of n in the table are those found by using logarithms; therefore, use of the graph may result in values of n being slightly different (but still within 0.1) from those listed in the table. Errors caused by the impreciseness of the graph rarely have a significant effect on the value of mn.

The problem of coal resources is studied next. In 1974 it was estimated that the U.S. coal reserves were about 4.34×10^{11} tons (*Coal Resources of the United States* 1974), and approximately 5.58×10^8 tons were consumed in 1975 (*Monthly Energy Review* 1978). Table 6 is completed by the students as a group.

Recent annual increases in usages have usually been under 5 percent; however, it appears this figure may increase to 10 per-

TABLE 5

Estimated Petroleum Reserves A	Annual Use Last Year x	Rate of Increase r	Doubling Time $m = \frac{72}{r}$	Amount Used in First Doubling Period $y = \frac{3}{2}(x)(m)$	$\frac{A}{y}+1$	No. of Doubling Periods n*	No. of Yrs. Fuel Will Last mn
6.46×10^{11}	2.17×10^{10}	0	**	**	**	**	30
6.46×10^{11}	2.17×10^{10}	2	36	1.17×10^{12}	1.55	0.63	23
6.46×10^{11}	2.17×10^{10}	4	18	5.86×10^{11}	2.10	1.07	19
6.46×10^{11}	2.17×10^{10}	6	12	3.91×10^{11}	2.65	1.41	17
6.46×10^{11}	2.17×10^{10}	9	8	2.60×10^{11}	3.48	1.80	14
6.46×10^{11}	2.17×10^{10}	12	6	1.95×10^{11}	4.31	2.11	13

* Where $2^n = \frac{A}{y} + 1$

** Does not apply

cent or above because of the shortage of petroleum.

The last few minutes of the second lesson are spent discussing some of the results obtained and the implications of those results. In particular,

1. if inflation increases at the present rate, the money system will probably need to be replaced by another one prior to the year 2100;
2. famine, war, disease, and so on will not allow population to get to the level projected for 2515; and
3. as reserves of fuel become depleted, the fuel becomes harder to get, thus driving up prices and decreasing usage.

(The students were delightfully perceptive, and a couple of their comments stand out. One student who had solved the wine problem, after considerable effort, said of the procedure developed in class for solving such problems, "Boy, it's a lot easier that way." In a discussion about how inaccurate the estimate of reserves might be, the students deduced that even if there is twice as much of a resource available as the amount estimated, the fuel will be depleted after only one more doubling period. After considering this dilemma for a few seconds, another student said, "Changing the percent (rate of increase) is the only solution." That statement sounds a lot like the message energy experts have been giving.)

The second lesson is closed with a hypothetical example Albert Bartlett used in a talk the authors heard him give. In his example, there is bacteria in a bottle and this bacteria doubles in amount each minute and the bottle is large enough to last one hour. If this situation originates at 11:00 o'clock, then at

11:01 lots of room;

11:02 still lots of room;

...

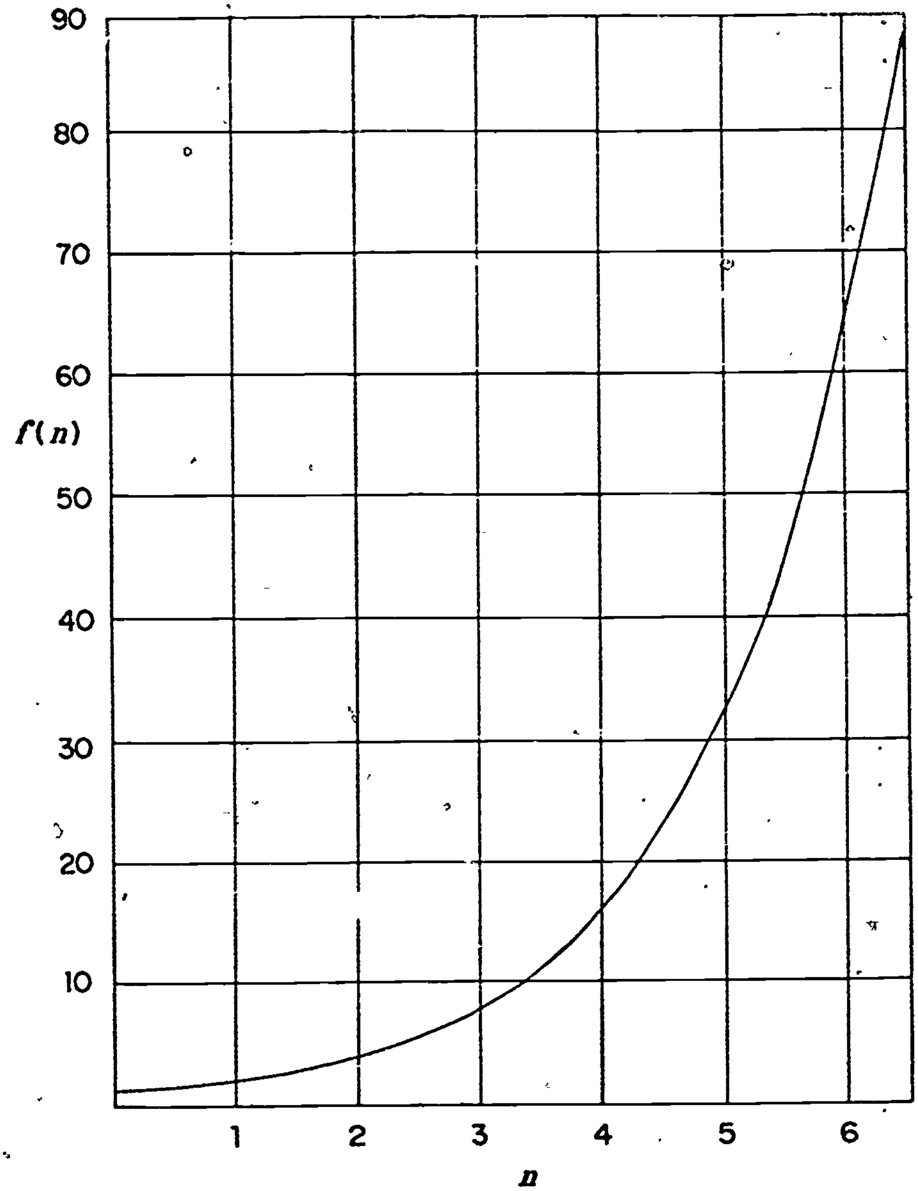

Fig. 1. Graph of $f(n) = 2^n$.

11:59 still lots of room
(only half full);

12:00 full.

Bartlett said that right now in the inflation, population, and fuel situation, it is 11:59.

Evaluations and Conclusions

As indicated earlier, the authors used these lessons with a variety of students (both college students and high school students) and found that the lessons were completed in about two hours. Portions of the tables could be completed in out-of-class assignments, thus cutting down on class time. For the most part, students who were capable of understanding compound interest seemed capable of understanding the lessons. Students did react favorably to the problems studied.

These lessons are recommended in that they

1. concern interesting and vital problems,

TABLE 6

Estimated Coal Reserves A	Annual Use Last Year x	Rate of Increase r	Doubling Time $m = \frac{72}{r}$	Amount Used in First Doubling Period $y = \frac{3}{2}(x)(m)$	$\frac{A}{y} + 1$	No. of Doubling Periods n*	No. of Yrs. Fuel Will Last mn
4.34×10^{11}	5.58×10^{8}	0	**	**	**	**	778
4.34×10^{11}	5.58×10^{8}	2	36	3.01×10^{10}	15.4	3.94	142
4.34×10^{11}	5.58×10^{8}	4	18	1.51×10^{10}	29.7	4.89	88
4.34×10^{11}	5.58×10^{8}	6	12	1.00×10^{10}	44.4	5.47	66
4.34×10^{11}	5.58×10^{8}	9	8	6.70×10^{9}	65.8	6.04	48
4.34×10^{11}	5.58×10^{8}	12	6	5.02×10^{9}	87.5	6.45	39

* Where $2^n = \frac{A}{y} + 1$

** Does not apply

2. relate mathematics to evaluation of social and scientific problems, and
3. give an example of a situation where use of a hand-held calculator makes the interpretation easier.

The authors suggest the possibility of investigation of similar problems involving the availability and use of other natural resources, such as aluminum and copper. Teachers could give students data on estimated resources and current usage patterns or ask students to find the information in the library.

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Section V

PROGRAMMABLE CALCULATORS

THE CALCULATOR IN THE CLASSROOM: REVOLUTION OR REVELATION?*

by

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THE CALCULATOR IN THE CLASSROOM: REVOLUTION OR REVELATION?*

I was an elementary school student in the late 1930's and attended the Ernest Prussing Elementary School in Chicago's far northwest side. It was a rather new school then. It had an excellent reputation for achievement and, as I recall, it was provided with the best of everything that could be provided during the great depression. We had one of the first "adjustment teachers" in Chicago and I remember taking a lot of tests that ultimately resulted in my skipping a grade. But what I noticed about the adjustment teacher (apart from the fact that she was an attractive young woman) was that she used a stopwatch and a slide rule. It was the first time in my life that I had seen such things. I remember that I had to build up my courage to ask her about them and she demonstrated them to me. I had no use for a stopwatch, but a ruler that could do arithmetic seemed like a miracle. I had to have one.

— Ted Stolarz

The microelectronic revolution is with us and the world will never be the same as it was before it started. That magic ruler that could do arithmetic is now obsolete. The hand-held pocket calculator can do everything the slide-rule could and more. Calculators can do arithmetic, algebraic, trigonometric, and statistical computations and do them faster than they can be done by the algorithms most students learn in school. What is more, they operate with an amazing degree of accuracy.

There are those who are disturbed by this and who feel that children should not be allowed to use calculators because if they do they will not learn the basic operations of arithmetic. The danger is there, of course, but the calculator when used widely under the direction of good teachers can generate interest in learning the facts and skills of mathematics. Once these skills are learned, the calculator, especially the programmable one, can take a great deal of the drudgery out of computation.

To illustrate this point consider the "ruler that could do arithmetic." The slide rule, which has been available for more than 350 years, makes a number of calculations quite easy. And, until the recent advent of the hand-held calculator, it was always standard equipment for engineering and science students. The same fear that today accompanies the idea of using calculators in the classroom also was prevalent when the slide rule began "taking over" the calculating process.

But the slide rule did not destroy students' ability to do fundamental operations longhand. It did not destroy their ability to do computation without it because to use the instrument skillfully, one had to know the process. The slide rule simply gave an <u>approximation</u> to the answer without some of the long computation.

Just as the slide rule did not destroy computational ability, the calculator will not do so either if used properly. Many facets of mathematics can be enhanced and supported by the use of calculators, especially the processes of estimation and approximation (for which the slide rule was often used).

complicated sequence in which it does one part over and over a set number of times or until a predetermined value has been reached and then goes on to do other things. It can store information in what are called memories and recall whatever is needed for use whenever it is needed. This means that the programmable calculator, unlike the ordinary calculator, can do sophisticated operations without needing the operator to initiate each step.

However, it does these things only if one can "teach" it to do so. One must make the calculator "learn" the separate steps in a sequence. Every mathematical step is possible on a nonprogrammable calculator. But the advantage of the programmable calculator is its ability to "learn" to do many operations with processes not possible on a nonprogrammable calculator. It is important to note that if one does not understand the process he is trying to program, the programing is a hopeless task. The programmer must identify and plan sequential steps for solving the problem as well as plan the order of arithmetic operations. Thus with the programmable calculator, even more than with an ordinary calculator, the thinking process of the user is primary if he is to build his own program.

Most of the applications of mathematics to real problems involve several operations. The programmable calculator permits us to address many different real problems without spending hours and hours in longhand computation. In addition, the use of the programmable calculator helps to develop skills in designing algorithms for the solution of problems which recur with different inputs. These skills will be as essential as basic arithmetic in the 21st century.

The programmable calculator or computer can also enable a person to use a special program (prepared by experts in a certain field) to do a complex operation he cannot do longhand. For example, there are statistical packages which can be very useful in providing results which are meaningful to the user even though he does not have the statistical training to do the computation or program the calculator.

The use of the programmable calculator in a creative way (i.e., having the user develop his own programs) is preferred over using the calculator or computer to run existing programs. Several examples of creative uses of the programmable calculator can be made. For instance, students can gain several geometric insights by using a programmable calculator. Suppose there are youngsters at the level of using basic geometric formulae for lengths, areas, and volumes.

Consider these problems for the student to explore:

1. How does the area of a circle of diameter d compare with the area of a square of side d? Try several cases and see if you can get a generalization. Look

at the formulae and see if the generalization makes sense.

$$A_c = \pi d^2/4 \quad \text{and} \quad A^s = d^2$$

$\pi/4 < 1$, therefore, A_c is a little more than 3/4 of A_s

2. What happens to the perimeter and area of a square, rectangle, circle if all basic dimensions are doubled? Tripled? Do the same for volume formulae.

3. If we have a rectangular box of fixed volume made out of sheet metal, what dimensions will make the area of the sheet metal a minimum?

4. You have a can in the shape of a cylinder made of sheet metal. If the volume is fixed, what radius and height will make the amount of sheet metal a minimum?

These problems can be quickly explored with the use of a programmable calculator. First, the student must know how to use the formulae required. Next he must tell the calculator how to use the formulae; that is, what sequence of steps it must take to arrive at a solution. Once this is achieved, the student experiments with different inputs to arrive at the final conclusion.

The programmable calculator should be introduced as soon as the child is faced with problems which occur frequently or which require more than a single step. This will happen not only in the computation but also in the application of mathematics to real problems such as interest, installment buying, cost comparisons, etc.

The use of the programmable calculator will not only make computation less tiresome, it will also allow the child to learn simple programming at an early date. Computers and their programs affect everyone in many ways. In the near future it will be as natural for one to understand how to use programming as it is now to understand spoken and written language. In the secondary schools it will become essential to give many, if not most, students some hands-on experience with microprocessors. The present generation of professional men and women, business people, and skilled workers has a handicap because the computer, its uses, and its languages were not a part of their early experience. Those who have had to learn this later have not had the advantage of an early introduction. Even more serious is the problem of those who must blindly use the results without any understanding of how they came about or, in the case of management, make decisions affecting computer operations with only a smattering of knowledge about what they do and how they do it. This generation, those in school now, must be given some background on this tool which they find everywhere in their lives.

It is obvious that the calculator, especially the programmable calculator, has much to offer mathematics students of any age. The following recommendations are made to promote and encourage the use of the calculator in the classroom.

It is recommended that:

1. Calculators be used in our schools.

2. School districts adopt a policy with guidelines in regard to the use of calculators.

3. Calculators be used as tools to reinforce the basic skills, not as substitutes for teaching the basics.

4. Calculators be used:

   to facilitate the learning of basic arithmetic skills at all levels,

   to facilitate the use of comparison in problem solving at all levels,

   to facilitate the use of estimation in problem solving at all levels, and

   to facilitate problem solving in real life situations at all levels.

5. Emphasis in calculator programs should be to teach children critical thinking. Calculators enhance critical thinking by:

   reinforcing the basic skills,

   helping in the basic skills of reasoning,

   reinforcing the thinking process,

   reinforcing problem-solving ability,

   promoting logical thinking,

   encouraging creative usage,

   providing stimulation and motivation,

   helping to develop number sequencing concepts, and

   aiding in discovering mathematical concepts.

All of this has important implications for teacher education. It is essential that those who will teach future citizens be prepared to introduce them to the tools which they will use and which will affect their lives in so many ways. It is suggested that elementary school teachers be required to learn how to use a calculator and be urged to learn how to use a programmable one. Secondary school teachers should understand how to employ the programmable calculator as well as the microprocessor or minicomputer.

The responsibility for this training will have to be assumed by the universities. Although there will be specialists who can teach the advanced material, most, if not all, teachers will have to have some background knowledge. At the moment it looks as though the universities will have to be sure that experience in the use of hand calculators, programmable and otherwise, plus experience in the use of microprocessors and main-frame computers is available to all who desire it. Further, it will be the responsibility of the universities to encourage all students to get at least minimal training with such equipment.

It is difficult to predict what the future will bring. The revolution in miniaturization, along with decreasing costs, will undoubtedly produce new wonders in the next few years. Whatever is made available in schools and colleges should reflect what is available on the market and in general use. The equipment purchased at the present should be minimal and consistent with good usage so that schools can take advantage of new developments. In short, we are entering a new age. It is hoped that in education we will enter it with enthusiasm and not be dragged into it reluctantly.

This document may be obtained from EDRS as ED 191 740.

THE PROGRAMMABLE CALCULATOR IN THE CLASSROOM

Theodore J. Stolarz

Introduction

I was an elementary school student in the late 1930's and attended the Ernest Prussing Elementary School in Chicago's far northwest side. It was a rather new school then. It had an excellent reputation for achievement and, as I recall, it was provided with the best of everything that could be provided during the great depression. We had one of the first "adjustment teachers" in Chicago and I remember taking a lot of tests that ultimately resulted in my skipping a grade. But what I noticed about the adjustment teacher (apart from the fact that she was an attractive young woman) was that she used a stopwatch and a slide rule. It was the first time in my life that I had seen such things. I remember that I had to build up my courage to ask her about them and she demonstrated them to me. I had no use for a stopwatch, but a ruler that could do arithmetic seemed like a miracle. I had to have one.

I saved nickels and dimes for what seemed an eternity and finally bought a wooden slide rule with "painted on" numbers and a little instruction book. The slide stuck in the rule like glue on damp summer days and fell out if you tilted it during the dry days of winter. I remember how disappointed I was when I found out that it could not add or subtract. But I was the only student in the 8th grade that had one and I carried it with me wherever I went.

Later on, at Carl Schurz High School an algebra teacher caught me using my slide rule on an examination. Her scolding made me feel as if I were "cheating." When I later found time to explain to her my reasons for using the rule she stated that it was a "crutch" and that I was doomed to going through the rest of my life carrying a slide rule. I had a nightmare where someone stole my rule and I was unable to multiply 3 by 4. The truth is, however, that she was partially correct. I have always had one with me until I purchased my Texas Instruments SR-52 calculator. My expensive Pickett slide rule now sits in my desk drawer at the college gathering dust.

Many educators, with good reasons, fear the use of electronic calculators by children in the elementary and secondary schools. I am sure that many of you have heard their arguments. They are often difficult to answer. Last year I was grading a problem one of my students handed in for a statistics course I was teaching. About half way through the problem her numbers started making no sense at all. I couldn't follow her logic. When I asked her to explain the logic of her solution she stared at the paper for a while and then said, "I think my batteries were getting weak." I confess to you that I did not know where to begin to help her. I was also worried that a student might some day flunk out of college because a transistor failed.

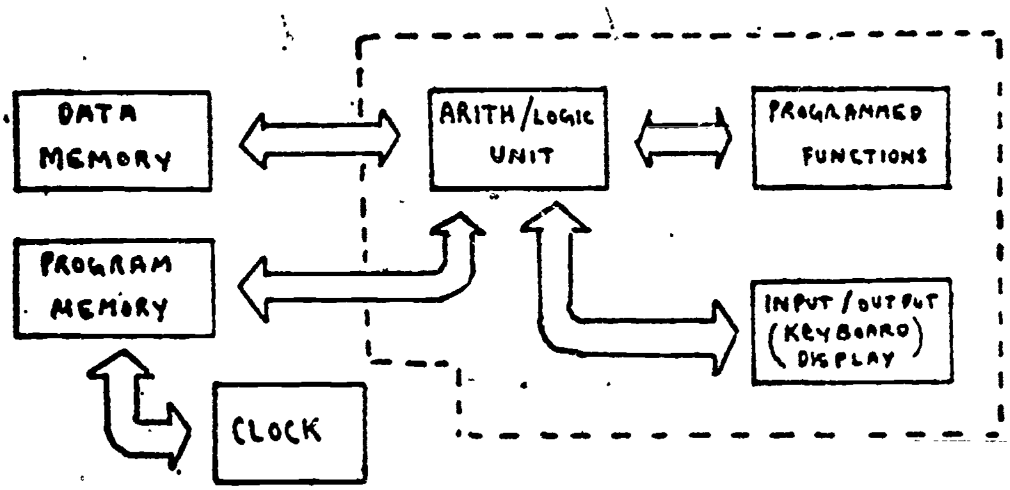
The Microelectronics Revolution

In reality, the "microelectronics revolution" is with us and the world will never be the same as it was before it started. It is by no means over. Whereas, in the past a teenager had to be able to add and make change to get a job we now find that in the "fast-food" hamburger shops the youngsters press buttons with pictures of hamburgers, milkshakes, and fried potatoes on them and the microprocessor in the register totals up the bill and indicates the correct change for the bills you offer in payment. Within a decade or less most college students and many high school students will have computers of their own with higher level languages such as BASIC or PASCAL and graphics capabilities. The calculator is now virtually taken for granted by children and versions of Texas Instruments "Speak and Spell" will be teaching vocabulary and spelling. The question is not, "Will we make use of calculators in the schools?," it is, "How will we do so?" In 1971 I purchased one of the first "hand held" calculators (the Monroe Model 10) for $350. A much better machine can be purchased at Radio Shack or K-Mart for about $10. I have in my home a personal computer system (the Heath H-8) which is more powerful than the "MANIAC" computer which was used to verify the mathematical equations for the first hydrogen bomb which was designed at Los Alamos, New Mexico, two decades ago. There is no turning the world back.

The Programmable Calculator

We are concerned here, however, not with calculators and their uses, but with programmable calculators. It is very important that we draw a clear distinction between these two because, despite their obvious similarities, they are really very different machines. A calculator simply performs operations that are familiar to the user and can be performed by the user without the calculator. The programmable calculator introduces a new mental process and a set of new concepts that must be learned. The programmable calculator is a computer disguised as a calculator. The "stored program" concept and the use of sequential steps following an "algorithm" to solve a problem is a computer process. This difference provides us with the opportunity to develop in our students a method of thinking that will prove to be of great value as they face the world of the future.

Let us examine this difference by reference to the diagram below.



A calculator contains the features shown inside the dashed lines in the diagram. All actual calculations are performed by the arithmetic unit which sends the results to a display (light emitting diodes or paper tape). Input of numbers (data) is accomplished using the keyboard. Certain keystrokes require the arithmetic unit to address preprogrammed subroutines which extract square roots, calculate logarithms or trigonometric functions, or in more complex machines convert degrees to radians, solve rectangular to polar coordinates, etc.. The analogy to the central processing unit, input/output functions, and subroutine branches is obvious to anyone with a passing acquaintance with computer architecture. The similarity is even greater when we recall the fact that all operations are actually done in binary code using such standard computer techniques as two's complement arithmetic and the rule of Boolean Algebra. All that need be added to build a complete "digital computer" is to provide a program memory which is sequenced by a "clock" which feeds the steps to the arithmetic unit at a speed within the limits of its capacity and a data memory which can store numerical data on a "destructive read-in, non-destructive read-out" basis. All programmable calculators have these features and they differ only in memory sizes, speed, provision of remote recorded program entry, and the "richness" of the instruction set provided to manipulate the various "blocks" shown in the diagram.

The key point is this. The logic or "mental process" used to program the programmable calculator is identical to that used to program any computer. In the calculator, programming is done by defining a series of "keystrokes" that will solve a problem for the user. In computer languages such as "assembler languages" or so-called "higher level" languages such as ALGOL, BASIC, APL, FORTRAN, COBOL, FOCAL, PASCAL, etc. we use a fixed set of written instructions in a rigidly defined syntax to solve the problem. The mental process used by the programmer is fundamentally the same. It involves developing an "algorithm" which breaks down the solution of the problem into a series of sequential steps which must be executed in an exactly defined sequence. There is no room for ambiguity in either case.

Planning Algorithms

The key to developing programs that will solve problems using computers or programmable calculators is skill in developing algorithms. The programmable calculator is ideal for introducing this skill because the user can concentrate on the problem of how to accomplish a goal without the added confusion of learning the syntax of a computer language. This is not immediately obvious. I had the unfortunate experience of doing all this backwards. My first efforts at building algorithms were with the IBM 1401 computer which had a FORTRAN compiler. I had little prior experience with computers and no instruction in their use. I acquired the IBM technical manuals and spent some weeks before I could get any idea of how to get numbers into the card reader and results out on the printer. During most of this time I had to operate the computer myself and the sheer size of the thing was unnerving. The few programmers around the computer did not know FORTRAN and kept trying to correct my syntax, not my strategy. Most of my errors were in program flow, not syntax. The compiler caught most of my syntax errors. The computer was faithfully doing what I told it to do, not what I wanted it to do.

After about a year of this experience, I was writing many successful programs, but my efforts tended to waste a good deal of computer time and printer paper. At about this time we acquired an early programmable calculator in the psychology department. It was made by the Monroe company and we had to punch octal codes into blue cards to represent keystrokes, which was somewhat confusing at first. But I found that solving problems with this calculator gave me much greater skill in writing FORTRAN programs for the large machine. No one prior to this took the time to teach me the fundamental process of designing efficient algorithms. I was too worried about losing control of the high-speed printer which could empty most of a box of paper before the computer operator could stop it or in counting card columns to construct input format statements.

Later on in this conference when we see demonstrations of problem solutions using programmable calculators we should keep in mind that the process we are using includes a series of concepts that are necessary for the efficient planning of algorithms. The programmable calculator can be used to teach and/or illustrate these as we develop this skill in our students. The following is a list (not complete, of course) of the kind of concepts I am referring to.

1. The use of a "program." The key concept which students may not understand initially is that of using a "program" to solve a problem. The programmable calculator has in common with the digital computer the fact that using a supplied program allows the user to solve a problem which he may not know how to give. Once a program is written it can be used many times. A program I wrote for test item analysis in 1971 is still widely used at our university (and in many other places) to evaluate multiple choice tests. It performs thousands of calculations using some rather sophisticated algorithms. Most users are unaware of the statistical niceties it contains. They do, however, understand the output it produces, and that is all that matters. This is the "magic" of the computer; programming languages are universal. Pre-written programs are available for all programmable calculators and they are very useful.

2. Planning sequential steps for problem solutions. While some texts start out with "flow charts" and diagrams which define an algorithm, the student should first learn a fact of life. A computer can only do one thing at a time. This is true of all computers, no matter how expensive or sophisticated. We can, by clever programming, make a machine seem to do several things at the same time. However, if we have one central processing unit through which all data must flow, the computer has the capacity of doing only one thing at a time. This is very evident when using a programmable calculator where the sequence of the keystrokes is important and the task must be reduced to a finite number of steps done in a certain order.

3. Planning the order of arithmetic operations. While I am sure that the teachers of mathematics present here today teach their students that an algebraic expression or formula is merely a convenient "shorthand" for specifying the order of arithmetic operations, some of my students act as if they do not know this. Planning the series of keystrokes needed to solve an equation can reinforce the student's understanding of the importance of this order. For example: $a + (b/c) \neq (a + b)/c$.

4. Memory storage and retrieval. Skill in manipulating memory is a mark of the good programmer. The concept of depositing intermediate results into a memory "bucket" for later use is not intuitively obvious. Less obvious is the placement of a constant like pi or e into a "bucket" for later use on a repeated basis. This is easier to see on a programmable calculator than on a computer using a language like BASIC where we might say, "LET X2=2.712828." The beginner does not see that the computer will reserve a fixed number of binary digits in memory to hold the value of the variable designated as X2. Most programmable calculators allow the user to add to a memory location or subtract a value from its contents directly. This is excellent practice.

5. Planning input/output operations. The output of a calculator is typically a number (or a series of numbers) on a lighted display or on a paper tape. The builder of the algorithm must plan when the program is to display these numbers and he must "document" his program so the user will be able to identify what the numbers mean. He or she must also plan the program so that it will pause so that data can be entered into the sequence via the keyboard so that program flow can proceed. This is a skill that has a positive transfer to the learning of computer programming in the future.

6. Branching forward or backward. While program execution is sequential the sequence does not always proceed in a single direction. Some parts of a program may be executed only one time while others may be executed many times to accumulate sums, products, quotients, etc. This is easily accomplished by "labels" in many calculators. Some calculators allow the user to set "flags" which can be tested to bypass "GOTO" instructions which branch programs back over previous steps. Practice with these techniques is very valuable.

7. Conditional branching. Most programmable calculators allow tests of the value in the "accumulator" or display register to see if it is positive, negative, or zero before branching to a portion of the program occurs. The test for "greater than, less than, or equal to" a certain value is easily accomplished. Such branching is so frequently used in computer algorithms that it is actually difficult to write a program of any complexity that does not use this technique.

8. Forming repetitive "loops." Using the above concepts it is possible to teach the construction of programs which "loop" through a series of steps for either a fixed number of times or until a desired result is accomplished. While this is planned for in programmables which have a "decrement memory and branch when zero" instruction, it can be done with any programmable which provides for the "+-0" branch test. Learning the concept of "looping" is difficult for many students who first attempt it with "FOR-TO" loops in BASIC or "DO" loops in FORTRAN. It is one of the most widely used techniques in the construction of computer algorithms.

A good exercise to teach this concept is to have the student write a program to obtain the square root of a number using the method of successive approximations suggested by Newton. This is how many computers calculate square roots.

9. Clearing storage. The fact that memory locations can hold "garbage" from previous program steps and must be "cleared" or set to zero if they are to be added to or subtracted from will become painfully clear after students write programs that "bomb out." I violated this rule the other day after over a decade of computer and calculator use and hundreds of successful programs in use all over the country.

10. Overlaying memory. It is an excellent builder of mental discipline to fit a large program into a machine that is limited to a small number of memory storage locations. Clearing and re-using memory locations builds good programming skills. I find myself sometimes getting careless in this regard when using my home computer which has 24,000 bytes of memory which will soon be expanded to 40,000 bytes. Starting with a large machine can develop bad habits.

11. Use of subroutines. Most programmables allow for construction of subroutines to perform repetitive chores with economy of program steps and memory storage. This is a concept that takes some practice before it becomes a matter of habit.

12. Indirect memory addressing. Some programmables allow for branching to a program step specified in a memory location. This value can be the result of calculations within the program. This is an advanced skill but it is within the capabilities of bright students. It provides a very powerful tool in the construction of some algorithms.

13. Concern for program length and execution speed. There are, of course, several ways to construct algorithms to solve any specific problem. All of them are "correct" if they produce the desired results. The "best" algorithm is the one that provides the result using the fewest program steps or keystrokes. Since the "clock" in a programmable calculator is relatively slow as compared to a computer "clock" this speed difference is more obvious when using the calculator. The shorter program is also easier to load since there is less chance for error.

The list is not complete. However, it does suggest how much a person can learn from the use of a programmable calculator. The little machine that fits into a pocket or purse is in fact an extremely complex marvel which contains thousands of electronic components arranged so that they can be instructed to solve a myriad of problems. In at least this one area, that of teaching the building of algorithms, its use in the classroom is very valuable. The positive transfer of these learnings to future learnings should be obvious.

Other Uses[1]

While up to now my interests in computer algorithms, programming languages, and digital electronics clearly have influenced my presentation of how programmable calculators can be used effectively in teaching in elementary and secondary schools, there are many other uses that can be found for them that have educational value. It is a feature of this conference to explore these potential uses through the exchange of ideas. The shared creativity of many skilled educators can open up avenues and approaches that could not be foreseen by a single person regardless how creative he or she may be. Again I must reach into my own experiences which merely suggest how fruitful the field is. I will illustrate a few ideas that I have. In most of these the teacher does the "programming" but there is no reason why a student or a group of students could not be involved in this effort.

1. Curve plotting. In teaching the concept of the "normal curve" to my introductory statistics students I try to convey the idea that the curve has no "standard" shape but is a plot of an equation involving certain constants and variables. The constants are pi and e and the variables are the mean, the number of scores, the standard deviation, and the individual scores in the distribution. By writing a program for the calculator which solves for the height given the values of the variables previously mentioned, I can plot the curve several times to show what effect the variables have on the shape of the curve. Solving

$$-(X-M)^2/2S^2$$

$$h = \frac{n}{S\ \overline{2}}$$

for h given n, S, M, and X fifty or a hundred times is a prospect no man can face with equanimity. A short program for my Texas Instruments SR-52 gives the values of h in a few seconds and allows the curve plotting to go on. In a secondary school class the "feel" for simple equations employing powers of 2 or 3 could be of value. It might also be useful to plot lines by selected points in linear methods where a program for a "least squares" fit of a line to a set of points is easily written.

2. Building tables. I have written simple programs using the formulas for compound interest to determine how much I must save, given current bank interest to have enough money to purchase additional memory or peripheral devices for my computer at a certain point in time. I also calculated depreciation schedules for the SR-52 for my income tax form. I can see possible uses for this kind of programming in teaching business mathematics or even consumer education to students who will be users to credit in a few years.

3. Science education. I received a "lunar lander game" program with the SR-52 and I spent a few hours playing with it. Most of the time I impacted the moon at various velocities and theoretically killed myself several times. I can see, however, that the acceleration of a falling body could be demonstrated, or the trajectory of an artillery shell estimated. The ease of plotting data points gives a better feel for the dynamic nature of what is happening rather than the static feel one gets when it takes considerable time to establish a single data point. I have also used the metric conversion programs when my favorite scotch suddenly appeared in half-liter bottles and I wanted to calculate the price per fifth gallon.

4. Mathematics classes. The combination of the availability of trigonometric functions and programming capability helped me verify the correct operation of a "Biorythm Plotting Program" I placed on my home computer and the computer system used by our university. The sine function is used to generate the curves, of course, but verifying the plotted points in the computer program helped "debug" the operation of the program and ensure that the leap years were handled properly. I can see many possible uses for this kind of programming in courses in algebra, geometry, solid geometry, trigonometry, etc. My lack of familiarity with the secondary curriculum in these areas, of course, limits me in anticipating uses.

I will leave the rest to you and I am eager to see how much I will learn before tomorrow is over. Thank you for your kind attention and I hope that you will find this conference to be of value in our common goal of improving the mathematics skills of our students.

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THE CASE FOR PROGRAMMABLE CALCULATORS by JOHN J. WAVRIK

With all the attention being given to computers these days, another quite remarkable instrument often gets overlooked: the programmable calculator. I don't wish to open a debate about whether or not programmable calculators are computers. Let's assume that they are not. A programmable calculator has many computer-like features, but it is basically limited in its input and output: it accepts numbers and outputs numbers. Most computers can process words as well as numbers. This can be very nice for games (in one Star Trek version, if you accidentally destroy your own base, Mr. Spock will print you a message telling you that what you have done "is not only illogical, it's stupid!") and for non-numerical information processing tasks. Some computers can produce graphics displays, some can be used to control other machines. Calculators will only calculate. At the present time, though, they are better at this than modestly priced computers. If "number-crunching" is your game the instrument of choice is a calculator rather than a modest computer. If factors of cost and portability are thrown in, we find that calculators form a distinct breed of machine that has its own right to exist.

At the moment I find myself heavily involved with numerical work. I do have access to a large computer system but find myself doing much of the work with a programmable calculator. It may be a personal quirk, but I find it easier to devise and test algoritms using the calculator. In some cases, though, the results that really interest me require more storage and speed than the calculator can provide. In such cases the procedures I use are designed with the aid of the calculator but the computer does the final work. It is true that computers run more quickly than calculators. Be sure, however, to count the time from posing the problem to obtaining the solution! In many cases the computer's intolerance for misplaced parentheses and omitted semi-colons will make the debugging process far more time-consuming. If you work with a computer center, do you have to drive there to drop off a deck of cards and come back later for the results? How long does it take to enter a program and data into the machine? Of course, if you're involved in long range weather forecasting a calculator won't do (but neither will a $600 home computer!).

There is another area in which programmable calculators are useful: education. I am working with a mathematics club for elementary school students in grades 4—6. In an effort to convey to students what is involved in getting a computer to do what it does, I demonstrated some simple programs with a programmable calculator. To my surprise, several of the students showed readiness to learn programming. Writing original programs is an excellent problem-solving task for elementary school students. A pair of fourth graders who decided to write their own perpetual calendar program not only had to do some research into calendars and astronomy, but they also had to solve several technical mathematical and programming problems. The calculator is also useful in allowing the exploration of areas that would be less accessible otherwise. A sixth grader, for example, used a random number subroutine to conduct experiments in probability and statistics. The use of the calculator also has provided insight into computers. Students have a better appreciation of what is involved in computer programming and of what computers can and cannot do. Some have found it quite easy to learn a computer language after their experience with the calculator.

Learning doesn't take place only at school. Indeed if a student has a great deal of interest in a special subject (like mathematics) it is unrealistic to expect any school to provide the type of instruction such a student really needs. Mathematically talented students have needs, interests and abilities that no mass education system can cope with. Such students should expect to do a great deal of their learning by pursuing their interests at home. To parents of these students I would seriously recommend the purchase of a programmable calculator. At a cost of about $100 one can buy what amounts to a personal laboratory for work in mathematics and computer science.

Dr. Wavrik is an Associate Professor of Mathematics at the University of California at San Diego. He is the author of "Finding the Klingon in Your Calculator", CALCULATORS/COMPUTERS Magazine, January 1978.

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A SUMMER COURSE WITH THE TI 57 PROGRAMMABLE CALCULATOR

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Let me wish that the calculating machine, in view of its great importance, may become known in wider circles than is now the case. Above all, every teacher of mathematics should become familiar with it, and it ought to be possible to have it demonstrated in secondary instruction.

—*Felix Klein*

(referring to the newly invented mechanical calculator) in an address to high school teachers, Gottingen, 1908.

In the few years since its appearance on the market, the programmable calculator already has opened up an entire new dimension in numerical exploration. Hundreds of problems from number theory, algebra, and calculus, which until now have required the use of computer facilities (and hence the knowledge of a programming language), can now be programmed and run on one's own pocket calculator at home, on a trip, or while on vacation—providing many hours of reward and virtually an endless variety of mathematical topics to explore.

Therefore, when I was asked by the newly founded Eau Claire Association for High-Potential Children, Inc. to plan a special mathematics course as part of its summer program for 1978, my instant choice fell on the programmable calculator. The proposal that subsequently evolved called for a six-week, three-times-per-week course based on the Texas Instruments TI 57. The chief philosophy behind the proposed project was to offer an enrichment program for students of the upper elementary and junior high school levels that would enhance their interest and motivation in mathematics; consequently, the syllabus put stress on ideas and concepts, rather than on techniques and rote learning. The course was to be one of fifteen courses offered by the association for its summer program, ranging in topics from biology and astronomy to art, Latin, and Greek mythology.

A grant enabled the association to purchase seventeen calculators, which in turn determined the number of participants in the program: the plan called for every student to have his or her own instrument, thus eliminating some of the frustrations that are common at computer terminals where many users have to share the same facilities. Two sections of the course were offered—one for ages 8–11 and the other for ages 12–15. There were no entrance requirements, but the intention was to offer the course to particularly gifted and motivated children, and this indeed turned out to be the case.

The TI 57 programmable calculator was chosen chiefly because of its simplicity of operation. It has a modified algebraic logic known as AOS (Algebraic Operating System), which not only enables algebraic expressions to be entered from left to right exactly as written, but also "recognizes" the hierarchy among algebraic operations. If, for example, we press the keystroke sequence $1 + 2 \times 3 =$, most calculators will display 9 as the result, because they perform the instructions sequentially as they are entered, always completing the previous calculation whenever a new operation is encountered. The TI 57, on the other hand, will display the correct result, 7, because it "knows" that multiplication must precede addition. This feature was considered to be of major importance; because of the age level of the participants it was very important to make the arithmetic as simple

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as possible and to avoid any complications due to different "logics." Also, the programming features of the TI 57 are just about right for the level of the students involved; the instructions are simple and straightforward, eliminating the necessity of learning any sophisticated programming language. This meant that we could "go straight to business," concentrating on the mathematics itself from the very beginning of the course.

Perhaps a brief description of the programming capabilities of the TI 57 would be in order. The instrument looks very much like any other scientific calculator, but a special key denoted LRN ("learn") transforms it into a "student" ready to receive instructions from us, the "teachers" (this attitude, that the children are really the teachers of their own calculators, was maintained throughout the entire course). The calculator is capable of receiving up to fifty program instructions, which is more than sufficient for most elementary purposes. A "pause" key instructs the calculator to display any intermediate results during program execution, whereas the R/S ("run-stop") instruction halts the program at any desired step. There are two decision keys, $x = t$ and $x \geq t$, that compare the current number x in the display-register with a prestored number t in the test-register and branch the program according to the outcome of the comparison. Branching can also be affected through the GTO ("go to") and RST ("reset") instructions; the former diverts the program to a specified location ("label"), whereas the latter sends the program back to the beginning. There are several editing keys that enable one to make corrections or changes in the program without keying in the entire program again. The instrument has eight memories, in which numbers can be stored, added to, subtracted from, multiplied by, or divided by, as well as exchanged with the display value. Thus, the calculator has full "memory arithmetic" capabilities, a feature that we constantly exploited in our programs. The display itself has an eight-digit capacity, but all calculations are internally done with eleven digits that are then rounded off for displaying. Figure 1 shows the keyboard of the instrument.

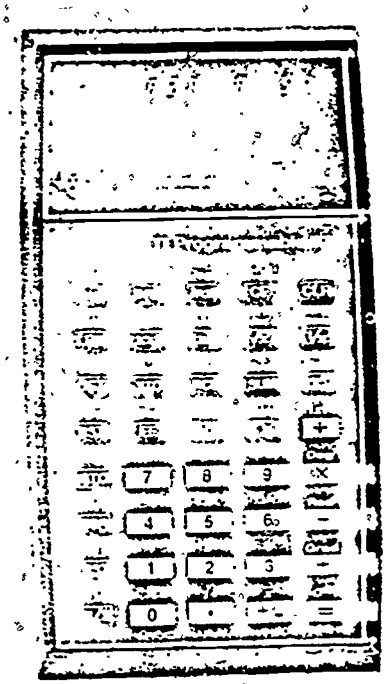


Fig. 1. The TI 57 programmable calculator.

Description of the Course

The course met for six weeks, three times each week, in the seminar room of our mathematics department. There, surrounded by books and periodicals, the pupils were exposed to some of the flavor of academic life at a university. They were free to check out any books they wished, and one of them asked to take home the thirteen books of Euclid! Whether he actually studied them I do not know, but the incident shows something about the curiosity of these students to learn and explore new ideas in mathematics, a curiosity that persisted throughout the entire course.

Each topic of the syllabus was used as an "excuse" to introduce a new concept, bring in some novel idea, or point out some highlights from the history of mathematics (table 1). In this way the students were exposed to the spirit and flavor of mathematics even without having the more sophisticated tools needed for a fuller grasp of these concepts. For example, the multiplication table was used to discover various "hidden" mathematical patterns, such as arithmetic progressions (any row or column of the table), the progression of squares 1, 4, 9, 16, 25, ... along the main diagonal, or the symmetry of the entire table with respect to this diagonal. The students themselves discovered these features in response to my questions, and they were fascinated to find out that the boring multiplication table has so many interesting secrets in it.

Next we learned how to "teach" the calculator to generate some of these patterns. For example, to generate the progression of squares we key in the following program ("2nd" means an upper-case instruction).

<u>LRN</u>
1
SUM 0
RCL 0
x^2
2nd Pause
<u>RST</u>
LRN
RST
R/S

Again, it was the students who suggested this program. When two different programs were proposed for the same task, we would write both on the blackboard and discuss their relative merits or drawbacks; if both programs worked properly, we would adopt the one requiring fewer keystrokes. Ultimately, some of the youngsters became so addicted to programming that they would come up with suggestions for making even a very short program shorter yet!

We then moved to arithmetic and geometric progressions and their sums. We mentioned some of the many cases where these progressions occur in daily life and nature—staircases, mile signs along a road, the countdown prior to a rocket launch, the petals of a flower, frequences of the musical scale, and compound interest, to name but a very few. We then recounted some famous anecdotes related to these progressions. The students were fascinated by the famous story of how young Gauss found the sum of the first 100 integers in response to his teacher's special assignment given to him so that he would not be bored in class. Even more fascinating was the story about the inventor of the game of chess: When summoned by the Shah of Persia and asked what reward he would like for his invention, he merely requested to have one grain of wheat placed on the first square of the checkerboard, two grains on the second square, four grains on the third, and so on until the entire board would be covered. The Shah, stunned by the modesty of this request, immediately called for a sack of grain to be brought in, but it soon became

TABLE 1

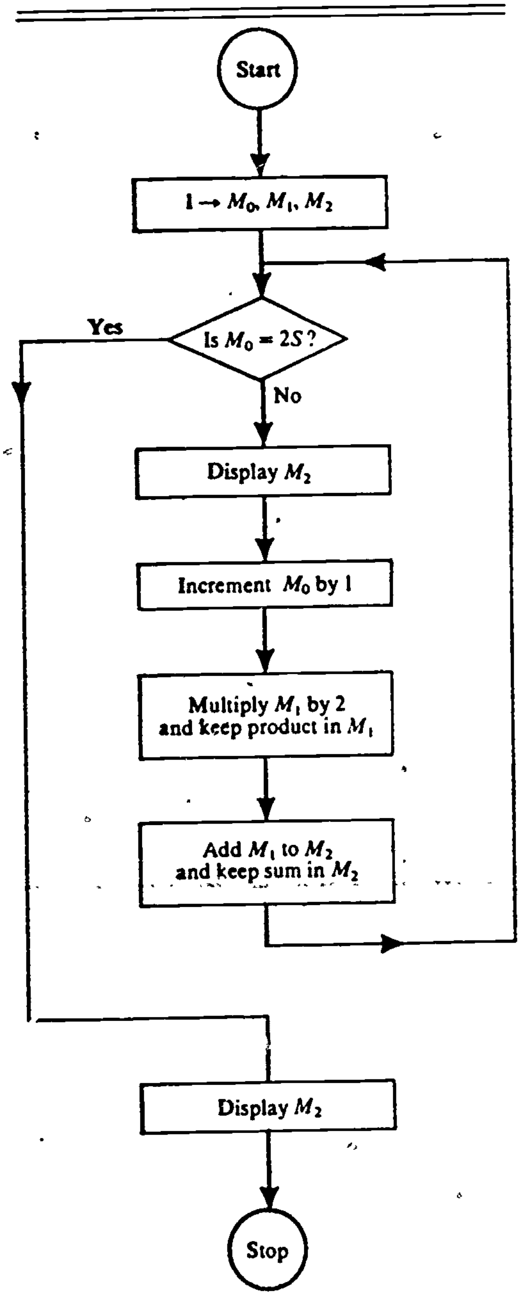
Topics Covered in the Two Sections

Section 1 (ages 8-11):
1. Introduction: how to use your TI 57 programmable calculator.
2. Teach your calculator how to count (counting by ones, twos, tens counting backwards).
3. The multiplication table on your calculator (all multiples of a given integer).
4. Sequences of numbers (all even numbers, all odd numbers, all squares).
5. Number patterns (sum of the first n integers, sum of the first n odd integers).
6. Arithmetic, geometric, and Fibonacci progressions.
7. The prime numbers.
8. Introduction to algebra (letter numbers, arithmetic sentences expressed algebraically).
9. Infinity and limits (limit of $(n + 1)/n$ as $n \to \infty$, sum of the decreasing infinite geometric progression).
10. Games on the calculator (nonprogrammed and programmed games).
11. The number π.

Section 2 (ages 12-15):
1. Introduction: getting acquainted with your TI 57.
2. A quick review of arithmetic.
3. What is programming?
4. Scientific notation: how to write large numbers.
5. Introduction to algebra.
6. Number progressions (arithmetic, geometric, and Fibonacci).
7. The prime numbers.
8. Other number patterns (Ulam's conjecture, happy numbers, Pythagorean triples).
9. Limits.
10. Newton's iterative method: finding the square root of a number.
11. Introduction to trigonometry.
12. The number π.
13. Calculator games.

clear that not even the entire grain in the kingdom sufficed to fulfill the task. (The name "chess," incidentally, is a distortion of the word "shah.") This beautiful legend never fails to fascinate novices when they learn how quickly the sum of the progression $1 + 2 + 4 + 8 + \cdots$ grows. We wrote a program that displayed the partial sums of this progression and stopped after the required number of terms had been reached. To make things simpler, we contented ourselves with a 5×5 checkerboard, so as to avoid entering into scientific notation, with which the younger class was not familiar. The program is presented here as a flowchart (table 2) and in keystroke form (table 3). In the former, M_i, $i = 0, 1, 2$ denotes the content of memory #i.

TABLE 2



The children were absorbed in anticipation for the calculator to halt the program after twenty-five steps and display the total number of grains: 33 554 431. (The program can, of course, be considerably simplified if one dispenses with the halt instruction.)

TABLE 3

LRN		
1	RCL 2	2nd Lbl 2
STO 0	2nd pause	RCL 2
STO 1	1	R/S
2nd Lbl 1	SUM 0	LRN
SUM 2	2	RST
RCL 0	2nd Prd 1	25
2nd $x = t$	RCL 1	$x \rightleftarrows t$
GTO 2	GTO 1	R/S

Another subject that generated great curiosity was the prime numbers. Most of the pupils had learned about the primes in school, but they were surprised to hear that some of the simplest questions about them are unanswered to this day: What is the next prime after a given one? (In other words, is there a prime-producing formula?) Why are there so many "twin primes" of the form $(p, p + 2)$? Are these twins finite or infinite in number? No one knows. We then programmed our calculators to generate the primes in increasing order (this program is not simple, so I gave it to the class). We let three calculators run this program continuously for twenty-four hours, and at the next session there was tremendous excitement as the children crowded around the three calculators in anticipation for the next prime to appear in the display. The highest prime we reached was 12 479. The TI 57 is so constructed that the action inside the machine dimly shows up in the display even without the "pause" instruction, and so we could watch the machine generating each integer and trying to divide it successively by all smaller integers. (One needs, of course, to test only the odd integers; also, it is sufficient to test the divisibility only up to and including the square root of the integer, a fact that considerably shortens the time of calculation.) Finally, we let the same program run also on the more advanced TI 58 coupled to the PC-100A printer, which enabled us to print all the primes. We let that program run uninterruptedly for about seventy-six hours, and the paper tape gradually began to fill the room. When we eventually stopped the program, it reached a total length of some fifteen meters, to the amusement of the children, reaching primes up to 30 000. We then carefully folded the tape and put it into a box for permanent storage so that no prime would ever be lost.

Another problem from number theory that we explored was Ulam's conjecture: (*a*) take any positive integer; (*b*) if the integer is even, divide by two; if it is odd, multiply by three and add one; (*c*) do the same with the new number, and so on; (*d*) ultimately you will reach 1. For example, beginning with 12, we get the following:

$$12 \to 6 \to 3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

(Of course, from 1 we can go on to 4, 2, and then 1 again; so we'll stop whenever we reach 1 for the first time.) That this is so for *every* positive integer was conjectured by Stanislaw Ulam (born 1909), but so far has neither been proved nor refuted by a counterexample. The number of steps it takes to reach 1 varies irregularly from one number to another and is difficult to predict; for example, it takes only 12 steps to arrive at 1 from 106, but 100 steps to reach 1 from 107, reaching numbers well into the thousands in the process. In programming this procedure into the calculator, we have to "teach" the machine how to distinguish between an even and an odd number. This is done by dividing the number *n* by 2 and taking the fractional part of the quotient (there is a special instruction, denoted Int, that takes the integral part of a number; its inverse, INV Int, will take the fractional part). This fractional part is then tested against zero and the program branched according to the outcome (for details of this program see Maor 1979). There was a great curiosity to test various numbers and find out how many steps it will take to reach 1 in each case (the number of steps was counted by the program and could be retrieved at the end). We even played a game with this conjecture, by choosing a number *n* and then letting everyone make a guess at how many steps it will take to reach 1; the one making the closest guess was the winner. The kids had special satisfaction in trying out very large numbers, such as 99 999 999 or 12 345 678 and watching the up-and-down sequence of intermediate numbers until the final 1 halted the program.

More than perhaps any other subject, the notion of infinity, with its related limit concept, enchanted the students and turned on their imaginations. It is here that the programmable calculator exhibits its full educational capabilities. The limit concept is one of the most abstract concepts in mathematics; and many students, even at the college level, are deterred by it. This is perhaps because one cannot visualize this concept in one's imagination and so it remains a meaningless idea. With the programmable calculator one can at once see how a progression tends to a limit as more and more terms are calculated. A simple example is the progression 2/1, 3/2, 4/3, 5/4, . . . , whose general term is $(n + 1)/n$. (There was a problem with decimal fractions, with which some of the younger students were not yet familiar; so I gave a very brief introduction to this topic.) The program for displaying the members of this progression is very simple:

```
LRN
-------
   1           ÷
SUM 0        RCL 0
RCL 0          =
   +       2nd pause
   1          RST
             -------
   =          LRN
              RST
              R/S
```

(This program can be shortened to nine

steps by writing $(n + 1)/n = 1 + 1/n$, but as most of the children were not fluent in algebraic techniques, I preferred the longer program). It was an exciting experience for the kids to watch the numbers in the display gradually approach 1; the question that loomed on everyone's mind was, Will they ever *reach* 1? With the calculator, of course, the answer is yes; when the difference between the current value and the limiting value becomes smaller than the smallest number the calculator can handle, the current number is rounded off to the true limiting value. But the students had no difficulty in understanding that this is so only because of the technical limitations of the calculator (as also of a computer); they all

agreed that in theory the number 1 can never actually be reached—only approached as closely as we wish. Thus, in one stroke, they grasped the true nature of this important concept. Other explorations with the limit concept were performed, such as the sum of the infinite decreasing geometric progression $1/2 + 1/4 + 1/8 + 1/16 + \cdots$, which was first explained in terms of eating one-half a cake, then another quarter, then another eighth, and so on ad infinitum. To the question whether one will ultimately eat up the entire cake, the class responded with a resounding no.

The last session of each week was devoted to playing games. We played many games, some of them nonprogrammable, such as Give-Take where the objective is to reach 999 999 by taking turns in adding and subtracting numbers from the display (see Schlossberg and Brockman 1976, pp. 20–21), others programmable, such as Hi-Lo or On Target (see "Making Tracks into Programming"). On other occasions we solved simple crossword puzzles, using the fact that certain numbers in the display, when turned upside down, form various letters and words, such as 14, which turns into *hi* (see Oglesby 1977).

In the upper section, algebra rather than arithmetic was stressed. At this age level the students are mature enough to use some of the tools of algebra and do some more advanced mathematics. First we discussed the so-called "scientific notation" for representing very large or very small numbers, using the EE key for this purpose. We designed a model of the universe in which the earth is represented by a cherrystone 3 millimeters in diameter (our own sun will then be a football-size sphere some 38 meters away). The class was then asked to place in this model the nearest star, 4.3 light-years away. This task made them acquainted with the notion of scale. After some struggling with the huge numbers involved, we finally came up with the answer: the nearest star would be some 10 000 kilometers away—about the distance from the equator to either pole! That gave the class some means to visualize the enormous distances among the stars in our galaxy.

An interesting topic from number theory is the Pythagorean triples. These are triples of integers (a, b, c) such that $c^2 = a^2 + b^2$—that is, (a, b, c) form a right triangle with c the hypothenuse and all other sides having integral lengths; (3, 4, 5) is an example. The Pythagoreans were fascinated by such numbers and attached mystical significance to some of them. The search for all such triples continued for many years until a procedure was found to find them: take any two integers (u, v) such that $u > v$; then the numbers (a, b, c) given by

$$a = u^2 - v^2, b = 2uv, c = u^2 + v^2$$

form a Pythagorean triple, as can easily be checked. (For a proof that this gives all possible Pythagorean triples, see Courant and Robbins, pp. 40–42.) It is easy to write a program that will display the members of the triple for any choice of (u, v), and the class was given this task as an assignment. I then asked them to modify this program so that even if by negligence we interchange u with v (i.e., $u < v$), the program will still work properly (this makes use of the exchange key). The students enjoyed discovering more and more of these triples. They soon realized that not all pairs (u, v) give essentially new triples; for example, $u = 2$, $v = 1$ gives the triple (3, 4, 5), whereas $u = 3$, $v = 1$ gives the triple (8, 6, 10), which is not essentially different from (3, 4, 5)—the two triples represent similar triangles. In order to get only essentially different triples, (u, v) must not both be odd and must not have a common factor.

The final two sessions were devoted to the number π. This number always carries with it a certain mystique to those who learn about it for the first time: why should as common and perfect a figure as the circle be related to such a strange number? We first discussed the fact that π can only be defined as a *limit*, which in turn means that we can only approximate it, never actually find its "true" value. We followed Archimedes' method of approximating the value of π by a sequence of regular inscribed and circumscribed polygons of more and more sides. This method requires a little elementary trigonometry, so we first defined the trigonometric functions sine and tangent. (These were very easily understood by all students: it must be said, in this connection, that the recent tendency of many textbooks to glorify trigonometry by defining first the circular functions, then the trigonometric functions, and viewing them as separate entities, has done a lot to complicate a subject that really is very simple in nature.) We then programmed the method into our calculators for polygons whose number of sides increases in a geometric progression beginning with $n = 3$ and doubling it again and again. The kids were fascinated to see the numbers in the display gradually approach the "true value of π.

We then examined some of the other methods available to calculate the value of π, some of which mark milestones in the history of mathematics, such as the Gregory-Leibniz series

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots$$

(see Courant 1947, pp. 318–19, 352, 440, 443) or Euler's series

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

(see Beckmann 1971, pp. 148–49; Maor 1976). The first series converges extremely slowly and is of no practical value in computing π (it is chiefly of historic interest, being one of the first applications of the newly invented integral calculus), but it takes a programmable calculator to actually measure the rate of convergence; it turns out that 628 terms are needed to find π to two decimal places (i.e., 3.14). The surprising fact is that the second series, which one would expect to converge much faster than the first (due both to the fact that the denominators increase as the squares of the natural numbers and the fact that all terms are positive), actually converges almost as slowly as the first, requiring 600 terms to get 3.14. On the other hand, the series

$$\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$$

and several others that can be obtained from the Fourier expansion of various periodic functions (see Courant 1947, p. 446) converge extremely quickly; writing programs for most of these series is very simple and straightforward.

We ended the course by having a little party, during which a pie was brought with the inscription "$\pi = 3.14$" on top, and everyone had a piece of it. Needless to say, the children enjoyed this pie, and there were many cheers and a lot of laughter.

The overall reactions to this first experiment were enthusiastic; participants expressed favorable reactions on evaluation forms, and many parents bought programmables for their children. Since then, five more courses have been offered, one of them a follow-up for students who had taken the first courses and who wished to do some advanced work with the instrument. Plans are also under way to form a calculator club where the students would meet once or twice a week, exchange ideas and programs, and test themselves on their own instruments.

Conclusions

Although the teaching of mathematics to young children by using the programmable calculator is an entirely new experience, it can already be said that it carries with it many promising potentialities: it enables the teaching of programming concepts without learning a detailed programming language, allows every student to use his or her own instrument (and take it home after class), and eliminates the usual commotion that can be seen at computer terminals. Last but not least, this course demonstrated again that even young children can learn and understand concepts from higher mathematics, provided that these concepts are presented to them at a level appropriate to their age. With its incredibly low price ($40-$50 currently), there is little doubt that the programmable calculator will have a marked influence on classroom instruction in the near future, and we may well see it change the entire pattern of mathematics education.

ACKNOWLEDGEMENTS

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Section VI

COMPUTERS

for

SCHOOLS

Classroom Computers: Beyond the 3 R's

Fred Bell

The classroom computer should, and can, go far beyond rote computer-aided instruction by teaching the student to analyze, evaluate and develop complex skills. Perhaps, as a result, the long-awaited "revolution in education" will be here sooner than predicted.

Since 1965, computer buffs, myself included, have been promising a revolution in education because computers are going to school. But where is this revolution? Certainly there has been at least a modest learning revolution; this is apparent from the many people who are learning about computers and using them to learn other things. In spite of the fact that some worthwhile applications are being done with computers in a few exemplary schools, this learning revolution has yet to take place in most schools. There has been an evolution in school learning (at least in many schools) that can be attributed, in part, to computer technology, but no real revolution.

Before considering the potential revolutionary effects of personal computers upon education, it is helpful to differentiate between school learning and out-of-school learning. The two are not always the same. We tend to learn things away from school when we want or need to learn them and we do so in our own way and at our own speed. This kind of learning has advantages and disadvantages. One advantage comes from higher motivation which encourages more inspired and efficient learning. On the other hand, the tendency to avoid difficult or uninteresting tasks may result in not learning some very useful and important things. Consequently schools are useful in coercing students, hopefully in a friendly and interesting way, into learning some things that are good for them which may not be learned otherwise. Out-of-school learning can be both good and bad, but so can in-school learning, which gets us to personal-computers and the education revolution.

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ersonal Computers and Dollars

One of the big reasons why personal computers may catalyze a revolution in our schools is that they are relatively cheap and should get even cheaper. Any family that can afford two color TV sets can now afford one color TV and a personal computer. Of course, any high school that could scrape up $10,000 per year for each of 10 years from a $1,000,000 per year budget could have had nearly all of its students using a minicomputer since 1969. (See James Saunder, **Mathematics Teacher**, May, 1978, pp. 443-447.) Fortunately, a family's decision-making processes in buying a personal computer are less cumbersome than a school's. Unfortunately for school students, as David Lichtman found (**Creative Computing**, January, 1979, page 48), educators are less enthusiastic about the computer's role in society and its potential for improving education than the general public.

But now, with low-cost personal computers, good computer applications may increase in schools. Home-computing enthusiasts have already begun to take learning out of the schools and are putting some of it back into the home where it belongs. Conversely, as more and more personal computers come to school, teachers can bring some of this good "street learning" back into the schools for the benefit of all students. Only $500 remaining in an equipment-and-supplies account at the end of the fiscal year can buy the first of many personal computers for student and teacher use.

History shows that many technological innovations that could be quite useful in promoting learning in schools do not get much use in schools until after they are common in homes and on the streets; for example, TV sets, audio recorders and hand-held calculators. Now that personal computers are "on the streets," we are beginning to see them filtering into schools. But will they be able to revolutionize education in schools? TV sets, audio recorders, calculators, and even minicomputers, while affecting what goes on in schools, failed to revolutionize education. Can we expect the personal computer to become a revolutionary agent? Yes, I think we can.

Personal Computers and Motivation

One of the most serious problems in schools is that of motivating stu-

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dents; that is, making them want to learn what teachers try to teach. The motivation problem occurs because sometimes teachers want to teach things (for good reasons) to students who do not particularly care to learn at the time. Perhaps the best hope for motivating students to learn in school is to pay attention to the nature of out-of-school learning. It appears that people learn non-survival things away from school for several reasons: (1) People learn to make things work (such as cars and computers) because they like to have control over impressive machines. (2) People learn to build model airplanes, radios, bookcases, etc. because they find satisfaction in creating something from nothing, or next to nothing. (3) People teach classes, give speeches, and write articles because they like to share their opinions and knowledge with others and possibly influence other people's opinions. Many things are learned because people enjoy the recognition and approval of other people. (5) Other activities that are not necessary for survival are carried out for relaxation, enjoyment, and self satisfaction.

But why do so many students dislike learning in school? First, students seldom have control over the academic machinery of schools; that is, the classroom learning environment. Second, creating and building tangible things occurs all to seldom in most classes. Third, students' opinions tend to be overshadowed by teachers' opinions in many classrooms. Fourth, many students get low grades in school, which interferes with their quest for recognition and approval. Fifth, much of what students have to do in school is neither relaxing, enjoyable, nor self-satisfying.

But how can a few personal computers in a classroom solve these motivational problems for students and teachers? Well, computers and computer-enhanced learning are not educational panaceas, but they can give students some real control over what they learn and how they learn it. Making a computer (an electronic monster) do one's bidding is fun for many people, in spite of the fact that it is, at times, tedious and frustrating. Writing a computer program and making it do what it is supposed to do is creating something — both a physical and an intellectual creation. Most people (including teachers and students) are impressed by good interactive computer games, simulations and tutorials, which provide recognition and influence for their creators. Finally, messing around, in a meaningful way of course, with a personal computer can be relaxing and enjoyable, in spite of many minor, temporary frustrations and aggravations.

Therefore, we find that personal computers in the hands of students in school can remove some of the artificial constraints of typical classroom environments and replace them with some of the personal freedoms inherent in many non-school learning situations.

Personal Computers and Learning

What is learned in school? English, reading, writing, arithmetic, French, history, etc.? Yes, these are some of the subjects that are taught but students should learn many other things that subsume all subjects. That is, students need to study each subject in a manner that permits them to function at all of the following cognitive levels:

knowledge
understanding
application
analysis
synthesis
evaluation
problem solving
knowing how to learn
creating knowledge

Schools are fairly good at imparting knowledge (i.e., "George Washington was the first U.S. president") and understanding (i.e., "2 + 3 = 5 because 2 marbles together with 3 marbles is 5 marbles"). However, schools are only moderately successful at teaching applications (out-of-school uses for each subject), analysis (breaking a skill or conceptual structure into its parts), synthesis (building complex skills or conceptual structures from simpler things), and evaluation (comparing skills and structures and making judgments about them). Schools and teachers have even less success at teaching students the skills and heuristic procedures of problem solving, how to learn independently of teachers and courses, and ways of conducting the research and explorations that go into creating knowledge.

During the past 15 years we have demonstrated, through many dramatic examples, that computers can be used in schools to help teach knowledge, understanding, and applications of various subjects—things that were being done fairly well without computers. This is the evolutionary aspect of computers in education. But what about the higher-level cognitive activities, those things that we haven't been able to teach very successfully in school? Herein lies the true power of computers (especially personal computers) to really revolutionize learning and teaching in schools.

Writing a computer program requires analysis and synthesis of the subject under consideration as well as the program itself. A student cannot write a program to tutor others, play a game, simulate a situation, or solve an exercise without analyzing the topic being studied and synthesizing it into a coherent teaching/learning program. The synthesis required in writing the program properly and the analysis in debugging it provides additional practice at synthesizing and analyzing. Since many non-tutorial computer programs are higher-lever applications of topics, the student programmer must evaluate the approriateness of alternative approaches to the topic and the program. When a student writes computer programs to extend and clarify topics in school, the six steps in problem-solving (posing the problem, precisely defining the problem, gathering information, developing a solution strategy, finding the solution and checking the solution) must be carried out. On the other hand, most so-called "problems" in textbooks are really exercises for practicing skills, which require only one of the six steps of problem solving; namely, finding the answer. After several years of working with people in Project Solo at the University of Pittsburgh, we found that many students and teachers could carry out independent research of their own choosing in computer-enhanced learning environments. That is, these people were creating knowledge and learning how to learn independent of people who were labeled as the teachers and rooms that were called classrooms.

Now personal computers can bring the Solo concept of high-level, self-motivated learning out of the research-and-development laboratory and put it in the hands of large numbers of students and teachers in school classrooms.

Carrying Out the Revolution

Even before the advent of personal computers (as early as 1972), the computer technology and courseware existed for a revolution in teaching and learning in schools. Now personal computers with their low costs, easy accessibility, total dedication to the user, and person-on-the-street popularity may provide the long-awaited catalyst that is needed to make some dramatic changes in how computers are used in schools. In a few years large numbers of students entering high school will be as familiar with a computer as they are now with a TV set, probably more so since they will have actively programmed a computer, in comparison to watching television passively.

As a consequence of the popularity of television, Americans are accused of having become spectators rather than participants in life. Personal computing certainly requires active intellectual participation on the part of the user. I have yet to hear of anyone dozing off while sitting in front of a personal computer.

For several years mathematics teachers worried about whether kids should be allowed to use hand-held calculators in school. The popularity of calculators outside school quickly settled that issue. Nearly every family had a calculator. Pre-school children played with them and students brought them to school. Teachers could not ignore calculators because it was impossible to keep them out of school; so now they are trying to determine how best to incorporate calculators and calculator-related skills into the school mathematics curriculum. Even if people try to keep personal computers out of schools, they are going to fail. In a few years, when they are more efficiently packaged and even less expensive, personal computers can fill the "lunch-box-technology" void created by school-lunch programs. Instead of a lunchbox, students will be carrying a PET or TRS-80 computer on a handle to school. When this time comes, an Apple for the teacher will really help a kid get a better grade in school. □

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GETTING STARTED IN A JUNIOR HIGH SCHOOL: A CASE STUDY

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Ramsey Junior High School has been making instructional use of computers for over a dozen years. Although our efforts at getting started seem like ancient history to us, we believe that our experience over the years may benefit those of you who are getting started today.

Ramsey has approximately seven hundred culturally diverse seventh- and eighth-grade students. About 41 percent of the students are from minority groups—mostly black. The family income levels range from very low to very high. For example, 32 percent of the students qualify for free lunch, and Ramsey receives Title I funding to provide instruction in reading and mathematics. In 1978 the student population, faculty, and school program moved from Bryant Junior High School to the Ramsey Junior High building as a result of declining enrollment and the city desegregation plan.

During the 1968–69 school year, the first computer teletype terminal was installed at Bryant Junior High. It was donated to the school by the nearby Bloomington-Lake First National Bank. Bryant was the first Minneapolis junior high school to have a terminal, and within a year all other junior high schools in the city were similarly equipped. Initially, the schools were furnished free computer time on a Honeywell time-sharing computer. In addition, time was purchased from Honeywell and other sources.

Today, Ramsey has two computer terminals that are used to access the Minneapolis Public Schools' Hewlett-Packard 2000 time-sharing system and the Minnesota Educational Computing Consortium (MECC) Control Data time-sharing system. These terminals are housed in a small computer room adjacent to the media center. The terminals are mounted on wheels, and there are telephone jacks in every mathematics classroom so that the terminals can be moved and used where needed. In addition, Ramsey had two Apple II microcomputers, one of which is located in Bill Robbins's classroom and the other of which is used primarily by the music department.

Computer users in Minneapolis receive external support from a computer resource teacher and computer teacher aide in the Minneapolis Public Schools as well as from a regional coordinator of the Minnesota Educational Computing Consortium. Inservice opportunities are provided by the school system, MECC, and the University of Minnesota.

In recent years, the authors served as director and assistant director of the COMPUTE project, which produced programs that allow teachers to have the computer generate tests and worksheets keyed to 500 different computation objectives. They also provided leadership for a similar project entitled Computer Generated Mathematics Materials, involving 1000 mathematics objectives in areas other than computation.

Personal Experiences and Observations of a Junior High School Teacher

I am dividing the subject of computer use in the junior high school classroom into five areas: (1) utility, (2) demonstration, (3) drill and practice, (4) learning games and simulations, and (5) programming. These areas are not mutually exclusive. I will use the term *computer* to apply either to a microcomputer or to a time-sharing terminal.

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Utility

There are many applications that can make life a lot simpler for a junior high school teacher, ranging from the very large library programs to the five-line program that you write on the spot. I use programs like Compute to make up worksheets. Other programs help me to change number bases, do modular arithmetic, or do operations on fractions. At the end of each marking period, I write a short program to average my grades for that period. I could do this on a calculator; but the computer automatically does the adding, dividing, rounding, and incrementing for extra credit. It even assigns letter grades. It also gives me a printout that I can use to check for errors. Teachers who use computers find that ideas come rapidly for a variety of applications where computers can make their work easier and less tedious.

Demonstration

Where the computer has a video monitor (TV screen) for output, it can be used for classroom demonstration. I usually connect two or three monitors to my Apple so the entire class can have a clear view. I have learned that pie charts, decimal-fraction equivalents, graphs, or just about anything that can be drawn on a chalkboard or overhead projector, can be quickly displayed on the screen—usually in a more dramatic, interesting, and dynamic manner. There are volumes of library programs available. (Most of the library programs that we use are available on Apple II diskettes. Persons wishing to purchase these programs should write to the Minnesota Educational Computing Consortium, 2520 Broadway Drive, St. Paul, MN 55113, for information.) In addition, I like to write programs that are specifically suited to my style of presentation. This sometimes takes a lot of time; but once the program is written, it can be used repeatedly.

Drill and practice

A computer is ideally suited for drill and practice for Title I or other remedial instruction. Drill and practice can also be provided in a regular classroom setting. In a small class (fifteen students or fewer), with a computer in the room, each student can spend part of most periods at the computer, practicing needed skills. In larger classes or when the computers are housed in a remote location such as the media center, students who will receive drill and practice on the computer have to be selected according to their needs.

Learning games and simulations

Computer learning games and simulations have high motivational value. For some students this can be the beginning and the end of computer experience. Every day I hear a student say, "I want to play the computer." This may be disturbing to anyone who is concerned with computer literacy. However, I have observed that the computer can be used as a motivational tool in any subject area. Many of the simulations that are available on microcomputers or time-sharing systems have educational value in mathematics, social studies, language arts, science, and other areas. (An example of simulations is "Lemonade" in which students simulate running a lemonade stand. A number of simulations are available from the Minnesota Educational Computing Consortium.) Moreover, for some students, playing games on the computer leads to serious programming later on.

Programming

The main obstacle to teaching computer programming in the classroom is usually the lack of availability of computers. One would like to teach a three-week or five-week or x-week unit in programming, like any other unit. Unfortunately, to accomplish this effectively in a class of thirty students would require the availability of at least ten computers.

The Minneapolis Public Schools are in the process of purchasing sets of microcomputers that will be loaned to schools for specified periods of time. In the meantime, I have found that I could get by with a limited number of computers by teaching programming concurrently with other topics. I introduce an aspect of programming, assign a project, and continue to teach the other mathematics topics as usual. Each day individual students go to the computer and type in their assigned programs while the others are doing their regular lessons. If a student makes an error, it will become obvious from the computer output. This is much better reinforcement than if I were to correct the program and point out the error before the student goes to the computer. When all the students have had an opportunity to complete the assignment, I interrupt the regular lesson for a day and introduce a new aspect of programming and assign a related project. This is a satisfactory solution to a high student/computer ratio. I do not like a batch method. because I feel that interaction between the student and the computer is an absolute imperative.

Some of the concepts I teach are care of floppy disks, loading and running programs, print statements, input. go to, if-then and read-data statements, for-next loops, and microcomputer graphics. I start with simple assignments in which students begin by writing programs that merely print their names and progress to assignments that involve problems, generation of tables, or creation of pictures using graphics capability.

My experience has been that after five or six computer lessons the novelty wears off, and some of the students become less than excited about computer programming. This reaction can be expected. However, each year there are several students who are enthusiastic. These students will generally continue on their own, with my help, by writing a simple game or modifying a game that is in the library. These students will be waiting by the door when I arrive in the morning, and they leave reluctantly when I lock up in the evening. Some of them have written amazingly sophisticated programs. In following up on these students, I have found that they do serious programming in senior high school and they tend to save up to buy their own microcomputers.

Tips for Getting Started

On the basis of our experience, we would like to offer the following suggestions for persons who are starting or expanding their use of computers in their schools.

Work cooperatively with others external to the school.

There is a distinct advantage to purchasing hardware that is similar to that being used by others in the area. Then experiences can be shared, software and programs can be exchanged, and cooperative in-service efforts can be arranged.

NCTM and its affiliated groups are including an increasing number of computer workshops in their programs. The Association for Educational Data Systems (AEDS) and its affiliated groups are another source of professional information (Association for Educational Data Systems. 1201 Sixteenth Street, N.W., Washington, DC 20036). Nearby colleges and universities can help by arranging in-service courses for credit. There is great benefit in state or regional computer consortia.

Build support within the school.

Most schools operate according to the golden rule: "Whoever hath the gold shall rule." Therefore, the support of the principal is crucial. One of the best ways to get this support is to show that there is a high degree of interest and support on the part of students and parents. Demonstrations and open house activities where students can show what they can do on the computer can build support of students, parents, administrators, and other faculty. Exclusive use of the computer by the mathematics department or by a single teacher within the department can narrow the support base. On the other hand, when teachers realize that the computer has applications in every discipline, the whole faculty can become a base of support for purchasing computers.

Be creative in seeking funds.

A local bank donated our first terminal

to us over twelve years ago. (Incidentally, it is still running.) Since computers are highly visible, they are particularly suitable as donations from businesses, PTAs, and other groups. It is a good idea to mount on each piece of donated equipment a brass plate with the name of the donor and the date of the donation. Various fund-raising drives can finance computer equipment. For example, at Ramsey we are using the proceeds from student fund-raising drives to purchase microcomputers. In recent years, federal Title IV-B funds, which are available to all public schools, are being used increasingly in many schools to purchase computer hardware and software. Title I funds are also sometimes used, but in this case the resources must be used for remedial instruction. Of course, there is always the regular school equipment budget. These budgets have been used in the past to supply the schools with overhead projectors, motion picture projectors, video equipment, and the like. By now, many schools are saturated with such equipment, so in coming years the funds might be better used to purchase computers.

Look ahead.

Computers are becoming increasingly useful in *all* subject areas as well as in guidance, media service, and administration. Make plans for increasing use of computers by students at all grade levels and ability levels. In the computer field, with rapidly accelerating change, flexibility is the key. Where possible, purchase hardware that supports a large variety of software that can be upgraded and that can run with various input and output devices. Most important, when you look ahead, be sure to provide your students with the computer literacy experiences that will prepare them for the world in which they will live as adults. For many years now in Minneapolis we have included questions on elementary BASIC computer programming on our eighth-grade criterion-referenced test, and we include computer literacy questions on our senior high school Basic Mathematical Knowledge Test. As you look ahead, plan for all junior high students to have hands-on experience and to write and run at least a few simple programs in BASIC.

Don't put off getting started.

This is a time of accelerating change in both software and hardware development. In such a time it can be tempting to wait for anticipated improvements and possibly lower prices before getting started. However, in times like these, such a strategy could cause you to wait indefinitely. Even though there are new developments around the corner, the hardware you purchase today should be useful for its entire physical life. An important reason for starting now is to gain experience and knowledge. Persons who delay get left further and further behind. There are more and more ways that instruction can be supported by use of computers, and you should make it a point to learn about them. Finally, you should get started so that you can help your students develop the computer literacy they will need to live in a computerized world.

GETTING STARTED IN A HIGH SCHOOL: A CASE STUDY

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Many teachers today are asking how they can get started teaching students about the computer, how they can motivate the students, and what kinds of activities they can try. In this article, I will relate my personal experiences in these areas.

I was a mathematics teacher in a junior high school before volunteering to teach computing. Along with another mathematics teacher and a science teacher, I joined the workshops for teaching computer literacy that were offered by Sylvia Charp, director of instructional systems of the public schools. Every secondary school was eligible to receive a computer terminal and support for a program of computer literacy. We were taught how to use the BASIC language and how to operate the terminal on a large Hewlitt-Packard timesharing system.

Our science teacher, John Di Lullo, played a major role in establishing computer literacy classes at our school and throughout the city by helping to develop *Computer Literacy: A Guide*. This hefty manual is currently in use and contains detailed lesson plans, sample computer programs and runs, sources of films, and more than sixteen hundred computer applications in a wide variety of fields. *Computer Literacy: A Guide* is out of print; however, an updated microcomputer version, *Computer Literacy*, is being published by the Pennsylvania Department of Education (Harrisburg, Pennsylvania 17108).

The other mathematics teacher, Joe Gariano, began teaching computer literacy to fourteen ninth-grade classes including academic, general, and practical-arts majors. There was one terminal in his room to serve classes of about thirty-seven students, which met one class meeting each week. There was a problem in getting each student hands-on experience.

When I began to teach these courses, the students did a lot of seat work on flowcharts and writing simple programs. Students were highly motivated and wanted to "talk" to the computer. I hung copies of their programs in the room. Someone discovered the Snoopy program that prints a calendar and the cartoon of Snoopy atop his doghouse yelling, "Curse you, Red Baron." This printout hung on our wall and copies found their way into the principal's office.

One program the students were assigned that caused excitement throughout the school was the rabbit program:

> An ecologist places a pair of rabbits on an island. The island is otherwise uninhabited and the food supply is plentiful. There will be no predators during this experiment.
>
> This particular breed of rabbits matures in exactly two months. At the end of the first two months, the female gives birth to a single pair of rabbits (one male, one female). Each pair of newborn rabbits takes two months to mature, and then each female gives birth to a single pair of rabbits. After reaching maturity, each female gives birth to a single pair of rabbits (one male, one female) at the end of every month. This continues without mishap for 36 months.
>
> Write a BASIC program that will compute and print for each of the 36 months the number of *pairs* of rabbits. At the end of the 36-month period, print the total number of rabbits present on the island. (From *Computer Literacy: A Guide*, Division of Instructional Systems, School District of Philadelphia, p. 168.)

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As you can imagine there were many discussions, arguments, and guesses about how many rabbits there would be. Many incidental concepts were learned along with new vocabulary. The answer, 1.49304E+07, generated more questions. We took time to develop their understanding of scientific notation.

Students enjoyed learning how to play some of the games from the library in the computer. Favorites were Tictactoe (the familiar game of Xs and Os with the students working furiously to beat the computer), Bagels (a logic guessing game in which the computer picks a three-digit number: the student guesses the number: and the computer tells whether each digit is in the right position, the wrong position, or not in the number at all—similar to the game Mastermind®), Guess (students guess a random number between 1 and 1000 chosen by the computer; the computer prints whether each guess is too high, too low, or correct and then prints the number of guesses taken), Hangman (the familiar word guessing game—but with the computer picking the secret word and printing out parts of the gallows with each missed letter guess), Speed Drill (a timed drill with whole number +, −, ×, ÷ problems), and Findit (a crossword puzzle generated from student input of words). In addition, problems for computer solution were assigned, and students could reserve computer time by signing a reservation sheet above the terminal. Time was available before, during, and after school. Several good typing students entered and ran the programs during their free period. They put comments on the papers when programs wouldn't run.

In 1971 I began to teach mathematics and computing at a new Parkway Program unit in Germantown. Naturally I looked for a terminal, only to find that it was twelve miles away. Since this school used community space, I looked in the neighborhood and hit the jackpot. Community Computer Corporation, a computer timesharing business, was owned by two educational boosters, Walter Friedlich and Ernie Philips, who donated the use of their meeting room to my basic mathematics and computer classes. What a great year that was with youngsters learning about computers from the inside of a computer business. Some of the programmers described what they were writing and also explained how students could use the terminals to program the same computer that was controlling the production of steel via telephone lines at a mill twenty-five miles away.

We used the CAMP textbook series (*Computer Assisted Mathematics Program*, Hatfield, Johnson et al., Scott, Foresman and Company, 1968) and *Basic Basic* by James S. Coan (Hayden Book Company, 1967) as resources.

Many ideas for programs came from the students. They wrote programs that printed

> Many ideas for programs came from the students.

tables of squares and cubes, the number of heads in x flips of a coin, and grocery bills and change due. They also learned to complete mathematics homework by writing programs. The class worked best in groups of three to five students.

Two years later we moved from the computer center to a regular classroom. Fortunately, we were able to get one terminal at Parkway. All my classes had an opportunity to interact with library programs and to write and run some of their own programs. One group of students wrote a computer program that printed "poetry" from the words input by others. Another group organized "speed drill" tournaments in which students competed to see who got the most problems correct from a set generated by the computer.

This past year we used an Apple II microcomputer. Students having difficulty with signed numbers in algebra could get some drill and instruction from Steketee's "Number Line" program (Steketee Educational Software, 4639 Spruce St., Philadelphia, PA 19139). Prealgebra students worked on programs to generate sequences and simulate a fast-food cashier. Students in the computer mathematics classes began by dramatizing parts of a computer and later programmed a robot (portrayed by one member of their small group) to rise from a chair, cross the room, touch the wall, return to the chair, and sit down.

Groups of four or five students rotate class-time use of the microcomputer.

Radio Shack let us borrow a color computer to use in class for a couple of days. During that time we saw students we didn't know were still enrolled in our course. Everyone wanted to make the computer draw color pictures and make music.

Groups of four or five students rotate class-time use of the microcomputer. When it's appropriate, they'll run a demonstration program for the class. Usually the program will include an input statement so that the class can interact with it and make hypotheses. First, students guess the results, then one team member enters the guesses and the output is discussed excitedly by the whole class. An elementary example is a short program that takes a student's input value and generates an output value for an equation like $y = 4x + 7$. Students try to guess the function. The best programs have, naturally, been those initiated by the students, although many texts offer examples (see references). The students keep me on my toes by their constant questions that begin with "How can I get the computer to do this?" Every day is different.

Last December some of my students and I attended a microcomputer fair at Temple University. They enjoyed working with the equipment and trying commercial programs as well as their own. Some did reports on the outing and on possible careers in the computer field.

I got started teaching about computers by volunteering for in-service training and then for teaching computer classes. There were no problems getting students motivated to learn; they were eager! You know some of the activities we found successful. Now it is your turn to get started.

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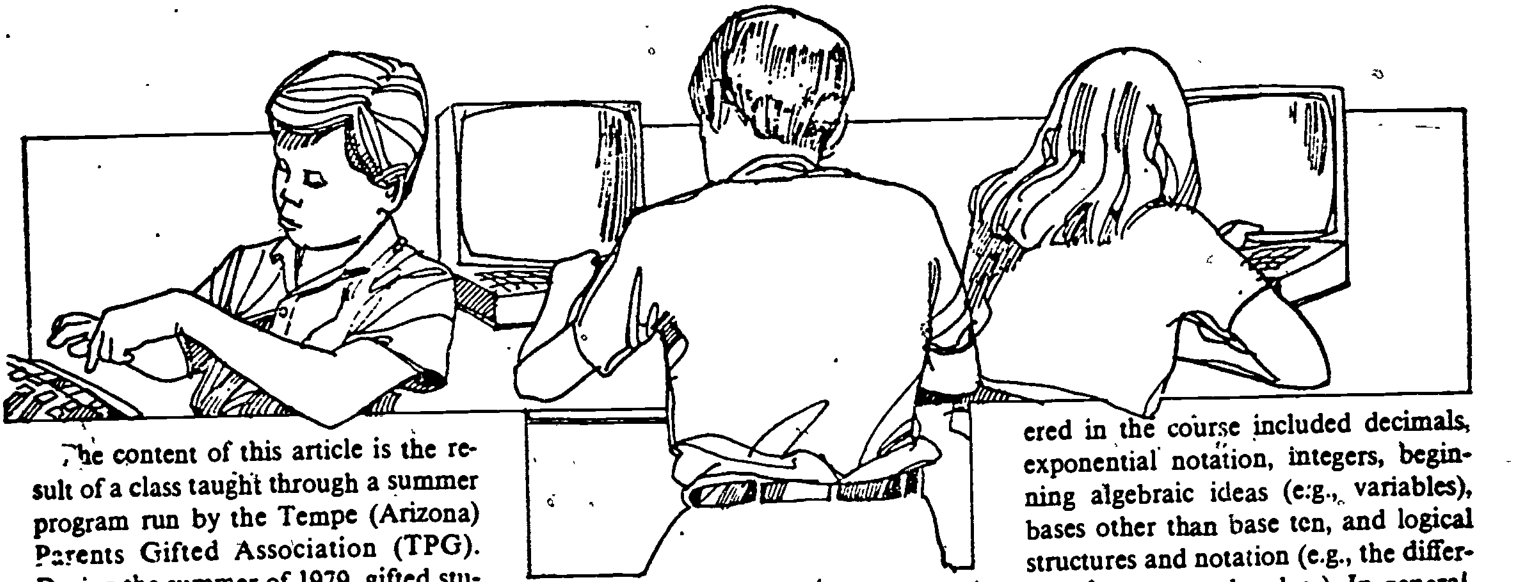
Section VII

COMPUTERS

and the

MATHEMATICS CURRICULUM

BASIC Programming for Gifted Elementary Students *By* James H. Wiebe



The content of this article is the result of a class taught through a summer program run by the Tempe (Arizona) Parents Gifted Association (TPG). During the summer of 1979, gifted students from the Tempe Elementary School District and other interested students from the fifth through eighth grades were allowed to enroll in a variety of classes. One option was a course in BASIC programming. Instruction in the class was based on the book *BASIC for Beginners* (Bitter and Gateley 1978).

Classes met three hours a day, four days a week, for a period of five weeks. Half of the instruction time was in related mathematical topics and half was in BASIC programming. Students spent about three hours a week entering and running programs on remote, teletype terminals connected by telephone line to Tempe Union High School's PDP microcomputer. Sixty-two students completed the course.

The discussion and recommendations that follow are based on techniques used with and observations from the fifth- and sixth-grade group. In most cases they are also relevant to the seventh- and eighth-grade group.

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Observations

Experiences with the 1979 summer school class in BASIC programming indicate that it is feasible to teach BASIC programming to average and above-average upper-elementary students. The programming structures that students are able to master, however, depend on the student's aptitude, maturity, and interest in programming.

Many students came into the course with no idea what it would be like. Their notions of computers were based on television programs, popular movies like Star Wars, and programmable electronic games. As a result some thought computer courses involved primarily activities and games with the computer. It was also apparent that some of the students were not prepared for an intellectual challenge, especially during the summer. The majority of the students, however, soon became involved in the class.

One might expect that many students in a gifted program would have received advanced instruction in mathematics. As it turned out, however, all students in the program needed supplementary mathematics instruction. For example, only a few of the students in the course had been exposed to either decimal numeration or integers. Supplementary concepts covered in the course included decimals, exponential notation, integers, beginning algebraic ideas (e.g., variables), bases other than base ten, and logical structures and notation (e.g., the difference between *and* and *or*). In general, students mastered these concepts very quickly.

At the beginning of the course, many students wanted to go run their programs immediately, before they had properly thought them through. It appeared that these students were not used to working problems through a step at a time. Rather, they were used to taking "intuitive leaps" to solutions. As the course progressed, however, students began to see the value of having their programs well designed and coded before trying to enter them, especially when they realized that they had only 45 minutes a session to enter, debug, and run their programs. Thus, an early course in computer programming might be good preparation for later work in mathematics requiring step-by-step solutions and proofs.

As for the type of assignments the students were asked to do, the children generally were not as interested in long, complex computer calculations or organizations of vast quantities of data as they were in having the printer do "neat" things. For example, they liked having the computer print out funny statements, using the INPUT command and having a friend interact with their program, having the printer print out sequences of numbers or words, and having the computer print out designs.

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Some students did not understand the more complex concepts presented in the course, such as imbedded loops and subscripted variables. It is not known if students would have mastered these topics if more time had been spent on them. It is possible that with a more leisurely instructional pace, with more examples and concrete experiences, these students might have mastered the concepts. In any case, students who did not understand the more complex programming structures and techniques seemed perfectly happy to run simpler programs of their own design and to enter and run more advanced programs used as examples in class (rather than write advanced programs of their own).

Recommendations

The following recommendations may be of value to persons planning to begin BASIC programming courses for elementary students.

1. Students should be provided with the necessary mathematics background before or during the programming course.

2. As often as possible, use concrete, pictorial, real-life examples of the concepts presented. Using mailboxes for memory locations, and having students walk through paths drawn on the floor to demonstrate loops are examples.

3. Use games and activities to reinforce concepts. "Computer Baseball" is an example. In computer baseball, students are divided into teams. When students come to bat, they are given a question. If they answer it correctly in a given amount of time, they go to first base; if not, they are out. The game lasts a predetermined number of innings. Questions may involve such things as the contents of a storage location after a given number of times through a loop.

4. Assignments should be multileveled so that the more advanced students are challenged without the slower students becoming frustrated. For example, more advanced students might be asked to develop their own unique programs involving loops as subscripted variables, while slower students are asked to complete and run programs involving the same concepts but using examples given by the teacher.

5. Introduce the PRINT and INPUT commands, string data, and mathematical functions early. Later introduce ideas such as loops and subscripted variables.

6. Assignments should be intrinsically interesting to pupils. "Real life" applications of the computer, such as data organization and scientific applications, do not appear to be as interesting to pupils as writing programs that draw designs, print funny stories, or print out a lot of data from a short program.

7. Have students flowchart and code programs before entering and running them.

8. Do not allow students to use their on-line time with "canned" programs. They should write and run their own programs.

9. To show the value of the computer and to keep pupils from using programming shortcuts, assignments involving mathematical computations should be too complex to be done by hand.

10. If pupils are sharing terminals, be sure to monitor their on-line time. If you do not, more aggressive students will dominate the terminals.

Assignments

Following are a sampling of assignments that upper elementary students might find interesting and challenging.

1. Construct a flow diagram for using a soft drink machine.

2. Run the following program from start to finish.

```
10 LET X = 256875
20 LET Y = 153097
30 PRINT X, Y, X + Y, X * Y
40 END
```

(* means multiplication)

3. Write a BASIC program to double $0.02 (2¢), 26 times. Before running the program, estimate what the answer will be.

4. Write a funny paragraph. Using the PRINT statement, write a program that will print your paragraph.

5. Write, enter, and run a program that prints your initial in a large block letter.

```
XXXXX       XXX       XXXXX
 XXXXX    XXXXX    XXXXX
  XXXXX XXXXXXX XXXXX
   XXXXXXXX XXXXXXXX
    XXXXXXX  XXXXXXX
     XXXXX      XXXXX
```

6. Use the IF-THEN and GO TO commands to write a program that reads a number (X) in a line of data (someone's height, for example), makes a decision, then jumps to and prints a statement about that number (for example, if X > 50, "YOU ARE TALL."

Draw a flow diagram for this program.

7. Write a program that someone else can run. Have the program ask the person some questions (use the INPUT statement). Use that information in a silly story. For example:

```
WHAT IS YOUR NAME?
JOHN
WHAT IS YOUR FAVORITE
ANIMAL?
DOG
WHAT IS YOUR FAVORITE
NUMBER?
7
ONCE UPON A TIME THERE
WAS A FAMOUS DOG
NAMED HORTENCE AND HIS
FAITHFUL SERVANT JOHN
WHO WAS 7 YEARS OLD....
```

Of course, your program should be longer and more interesting than this. After you have debugged the program, have a friend try it.

8. Run the following program.

```
10 LET X = 2
20 LET Y = 2
25 PRINT 'I AM AT LINE 25'
30 IF X = Y THEN 10
40 END
```

After 15 seconds, stop the execution by hitting the BREAK key. Draw a flow diagram for this program.

9. Write a program of less than 10

lines using IF-THEN and GO TO that prints out the numbers from 1 to 100 next to each other.

10. Write a program that—

   A. Prints out the following: "Central Control of Spaceship Enterprise to all crew members: All units have been checked and are ready to blast off for Venus. All personnel to stations. Secure safety belts. Ready."

   B. Uses the IF-THEN, GO TO statements and a counter to print out the countdown from 10 to 1;

   C. Prints "BLASTOFF";

   D. And prints out a picture of a rocket blasting off.

11. Write a program of less than 6 lines using FOR TO, NEXT to print the numbers 1 to 100, skipping every other number.

12. Use TAB (I) and FOR TO, NEXT commands to make a geometric design.

13. Write and run a program that uses the RND (X) command to simulate a coin-flipping game.

Conclusion

Although the class discussed in this paper was originally designed for gifted students, many of the students in the class had not been identified as gifted by the school district—they had signed up because they or their parents were interested. These students mastered many of the concepts and procedures presented in the class, successfully developing and running their own, unique programs. Thus is appears that many of the concepts and techniques in BASIC programming can be taught to the average upper-elementary-school pupil, especially those who are highly motivated.

Because of the present and future importance of the computer in our society, and because of the educational benefits derived from learning to program, we should consider making beginning programming experiences a standard part of the upper-elementary-school curriculum. A prerequisite is that prospective elementary school teachers, especially those who plan to teach mathematics at the intermediate level, become proficient in a programming language, preferably BASIC, and that they learn methods for teaching elementary school pupils to program.

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A Case and Techniques for Computers: Using Computers in Middle School Mathematics

By Larry L. Hatfield

Computers are rapidly becoming accessible to everyone. The costs of purchase have continued to decrease, with the recent miniprocessors and microprocessors representing pricing breakthroughs, and inexpensive microcomputers being promoted as personal, home computers. Though much of their suggested usages to date relate to family leisure or home management, some vendors offer packages for computer-based games and drills involving mathematical ideas. Instructional programs will become increasingly available as the marketplace develops. Today's middle school students are growing up in a computerized society. Students probably feel more comfortable (often excited and curious) with the prospects of "everyman" routinely using computers than do many adults, who still view computers as complex and futuristic.

Goals for Mathematical Education

We teach mathematics in a changing world. The discipline of mathematics and its applications are part of this continual evolution and our perspectives about the teaching and learning of mathematics must reflect this changing world. Our instructional goals and practices must be examined frequently to assure that we are providing for today's students an adequate mathematical education to function in the society of, at least, the near future. With this in mind, the following ideas about effective mathematics learning and teaching are summarized to suggest some viewpoints for considering possible applications of computers into middle school mathematics classrooms.

1. Mathematics learning is primarily a person-centered, constructive process: students build and modify their knowledge from experiences with task-oriented situations characteristic of mathematics. Students must experience opportunities and develop feelings of responsibility for revising, refining, and extending their ideas as the ideas are being constructed. Instruction based on the concept of constructed ideas will allow and expect errors or shortcomings in the student's responses at certain stages; the processes of identifying and correcting these are integral to the learning. *Computers can be effective instructional contexts for such constructive approaches to learning. For example, if the learning tasks involve writing or understanding a computer program, the student may be called upon to build up a procedure, test it, find the errors or inadequacies, correct or improve it, test it again, and possibly refine or extend it to a more general procedure.*

2. Solving problems is the most essential of all basic skills needed in our complex, changing world, and learning to solve problems that involve mathematics is the most fundamental goal of mathematics education. Studying the processes one uses in constructing a solution to a problem may be of great assistance in knowing generalized, perhaps heuristic, approaches to problems. *Various types of computer usages can feature mathematical problem solving and its study. Student-written computer programs can involve considerable problem solving.*

3. Most school mathematics curricula put a major emphasis on algorithms. Children learn not only computational procedures for various operations but also procedures for handling almost every mathematical task they encounter: finding solution sets, evaluating expressions, simplifying fractions, factoring, finding special numbers (e.g., prime numbers, greatest common factors, square roots, and averages), completing geometric constructions or transformations, renaming (e.g., fractions to decimals), graphing, checking (e.g., computational results), estimating, and so on. In most treatments the procedures are presented "ready-made." As a result, student learning of algorithms is often imitative behavior. If we believe that mathematics ought to be a sensible response to a reasonable situation, then more attention should be given to helping children to construct procedures. Helping students to identify limitations or revisions for their created algorithms can lead to significant thinking. *Computer programs are algorithms. Middle school students can learn to construct and use computer procedures for many mainline mathematical ideas. Such activity can further important goals related to algorithmic learning.*

4. Effecting quality learning is a complex combination of many factors. A reliable, knowledgeable source of

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help that generates, in a patient manner, interactive reteaching, practicing, and testing for each student, along with accurate records of student responses and cumulative progress, would greatly assist the harried classroom teacher. *Computers can be programmed to provide such assistance to the teacher, and teachers, as novice computer programmers, can readily learn to write and evaluate such computer programs.*

Instructional Computer Usages

Most of the exploratory usages of computers in school mathematics have occurred in the past ten to fifteen years. Major attention has been given to teaching elements of computer languages, such as BASIC or FORTRAN, and to engaging mathematics students in writing and using computer programs for various mathematical topics. This use of the computer reflects the obvious interaction of the programmable, numerical, "logical" machine and certain qualities or aspects of mathematics. Several other types of computer applications to teaching and learning also have been identified and explored. Let's briefly examine some of these usages of the computer before considering strategies for their implementation.

Programming

Mathematics students of all ages throughout the world are writing and executing their own computer programs. Mathematics teachers cite several purposes for students' engaging in such programming tasks: (a) building "computer literacy" through firsthand experiences with the capabilities and limitations of programmable machines; (b) reinforcing a taught (noncomputer) procedure through students' analyzing its steps and restrictions in order to construct a computer algorithm (program); (c) illustrating the use of a computer program as a dynamic problem-solving tool—even a poor program, as an active object, can be executed, "debugged," and modified; (d) providing transfer experiences through students' using their knowledge to "teach" the machine to generate the desired output;) stimulating high levels of student otivation and persistence toward more general, perhaps elegant, programs; and (f) emphasizing methods of solving, discovering, and generalizing by building programs, studying a program's output, and extending revised programs toward algorithms aimed at handling entire classes of problems.

Middle school mathematics abounds with situations that can be approached as programmings tasks: (a) finding solutions to open sentences; (b) testing for properties of number systems; (c) number theory topics—finding prime numbers, multiples, complete factorizations, greatest common factors, least common multiples, and deficient, perfect, or abundant numbers; (d) solving measurement problems—perimeter, area, volume, and angle relationships; (e) manipulating fractions and decimals—renaming, simplifying, performing the four arithmetic operations, and ordering; and (f) solving applied problems—finding averages, solving proportions, calculating percents, and figuring probabilities.

Practicing

Computerized practice has been extensively applied to school mathematics instruction during the past fifteen years. Of course, the fundamental purpose of a practice session at a computer terminal is to rehearse for more automatic recall or recognition of certain aspects of the ideas previously taught. Practicing programs usually do not deal with explanations of mathematical ideas. The role of the teacher is to develop the background understandings necessary for effective practicing to occur. Although of uneven quality, many practicing programs do exist.

Tutoring

Computer-based tutorial instruction involves the use of the machine to present an introduction or review of ideas which are not adequately known by the student. The stored program attempts to simulate a good tutor as it introduces, explains, characterizes, exemplifies, asks questions, accepts and evaluates responses, diagnoses difficulties, provides feedback and reinforcements, monitors performances, and selects appropriate placement into subsequent lessons. Tutorial programs are similar in structure and function to practice programs but often contain considerably more detailed text to be used in explaining the content of the lesson. They are usually more complex and costly to prepare and to offer than practice programs. Although much attention has been given to the experimental development of tutorial programs in certain federally-funded projects, this computer usage has not become widespread. With lowered costs for computers the prospect of such private tutoring may become more feasible.

Simulating and gaming

As a simulation device, the computer can often be used to deal with otherwise unmanageable phenomena. The advantages of computer-based simulation include the measurement and manipulation of variables that are difficult or dangerous to assess, the opportunity to experiment when it is not otherwise possible, the control of "noise" (irrelevant variables) that might otherwise obscure the item to be measured or studied, and the compression of the time dimension to allow long-term events to be studied in short periods of time. Among the contexts of possible interest to middle school mathematics teachers are situations that use probability models as well as other physical and social science applications of mathematics. The following are some examples of simulation programs: (a) probability (flipping coins, rolling dice, blinded drawings), (b) economics (playing the stock market, purchasing via loans, ruling a developing nation), and (c) sciences (effects and control of water pollution, piloting a spaceship, chemical or nuclear reactions, genetic manipulations).

Gaming programs are simulations featuring competitive settings in which one or more players can play, score, and win. Hundreds of computer-based games exist, with many appropriate to objectives of middle school mathematical education.

Testing

The computer has been programmed to serve as a test generator and administrator. Conceivably, each student could be given a tailor-made examination where the choice of objectives as well as number and difficulty of items for each objective would be specified for each student.

Fig. 1

```
1 REM Jason, Period 2, Ms. Davis
10 REM ** FIND ALL FACTORS OF A NUMBER **
20 INPUT "FACTORS FOR WHAT NUMBER", N
30 FOR D = 1 TO N
40 IF N/D <> INT(N/D) THEN 60
50 PRINT D;
60 NEXT D
70 GO TO 20
80 END

FACTORS FOR WHAT NUMBER? 12
 1  2  3  4  6  12
FACTORS FOR WHAT NUMBER? 35
 1  5  7  35
FACTORS FOR WHAT NUMBER? 59
 1  59
FACTORS FOR WHAT NUMBER? 57
 1  3  19  57
FACTORS FOR WHAT NUMBER? 101
 1  101
FACTORS FOR WHAT NUMBER? 864
 1  2  3  4  6  8  9  12  16  18  24  27  32  36  48  54  72
 96  108  144  216  288  432  864
FACTORS FOR WHAT NUMBER? 97
 1  97
FACTORS FOR WHAT NUMBER? 27
 1  3  9  27
```

Program notes:
This is an example of a structured, incomplete programming task.

Lines 1 and 10 are simply remarks, ignored by the computer during execution.

Lines 30 through 60 set up a "loop" where the set of possible divisors 1 through N are tested. The condition in line 40 tests for even divisibility, skipping on to the next trial divisor (line 60) or printing a divisor (line 50).

Using Computers in a School

To many mathematics teachers the prospect of incorporating the use of a computer into their curriculum appears as an educational "future shock." As a rapidly growing number of teachers in hundreds of schools are finding, however, the machines and accompanying materials are oriented toward the novice who has little or no programming and computer background. The computers are simple to operate, being quite "foolproof," and programming languages, such as BASIC, are easy to learn, yet powerful and flexible enough to satisfy the most advanced student. Numerous journals, oriented to the educational user of computers, provide further background as well as practical classroom suggestions and actual program listings. A variety of resource books for mathematics teachers provides detailed suggestions for developing student programming. Most computer companies sponsor educators' "users groups" that offer informative newsletters, meetings, and a clearinghouse service for exchanging computer programs among members. And professional associations, such as the NCTM, offer workshops at their meetings and written materials in their journals and supplementary publications to help teachers with computer-oriented activities.

A school typically begins a program of computer use with limited access to a computer, a single terminal or computer. Through workshops, teachers can learn to operate a minicomputer, run stored (ready-to-use) programs, and write and execute simple BASIC programs for several mathematical topics taught in the middle school curriculum. A sixth-grade teacher, for example, might decide to focus on two types of uses of the computer with his or her students during the first year of computer use, simple programming tasks and stored practicing programs. To develop the knowledge needed for students to write their own programs, the teacher might begin by discussing and demonstrating completed programs; later the students can be given incomplete algorithms which they analyze, complete, and execute. As students learn the BASIC language and how a program can be constructed, they will become more able to initiate their own programs for mathematical tasks.

The following is an example of student programming tasks for a particular unit to be studied:

The whole number system
(a) Find sums, differences, products, and quotients.
(b) Simplify number expressions.
(c) Make lists of pairs, given a function rule.
(d) Find factors (see fig. 1).
(e) Determine primeness.
(f) Find all prime numbers in a given interval.

Summary

Computers can be used in educational settings and for instructional purposes in more ways than are generally realized; they can be much more than a means to individualizing testing or drilling for competency in basic facts. Computers can be teaching aids that help to achieve the objectives for mathematics learning identified earlier in this article. When access to a computer is available, students will be able to use the computer for programming the solutions to problems; for simulating situations in order to test hypotheses; for gaming, as a study of probability and statistics; as well as for practicing, testing, and tutoring.

The purpose of this article is to help teachers become aware of the potential for this multiusage approach to computers.□

LET'S PUT COMPUTERS INTO THE MATHEMATICS CURRICULUM

There is plenty of room in the curriculum if we just drop the study of geometry.

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I propose a rather drastic restructuring of the traditional high school mathematics curriculum for those students who plan to go to college. Whereas this proposal is aimed primarily at the college-bound student, it should encourage everyone to study more mathematics.

The heart of the proposal is to delete plane geometry as a required course in the traditional academic sequence and to replace it with a year-long course in computer programming. My reasons for making this proposal are based on my experiences as a college teacher. Students often study only two years of mathematics in high school. They have usually studied first-year algebra and plane geometry. One year of algebra is grossly inadequate for the study of mathematics in college. Bear in mind that most college curricula require the knowledge of mathematics beyond the first-year-algebra level. For example, engineering and science majors require calculus. Psychology majors and sociology majors are usually required to take a statistics course. All education majors take some college-level mathematics, and business majors are often required to take a calculus sequence. By the time students get to college, many have forgotten most of the algebra they did learn.

I believe plane geometry is in the curriculum for historical reasons. I can recall reading about how the formal reasoning, the use of logic, and the mental discipline required to prove theorems in plane geometry would do wonders for the student's intellect. I don't think such claims have ever been proved. It is true that in the study of plane geometry, one is introduced to mathematical thinking, that one learns the rudiments of logic, and that one must be disciplined; but very few people ever go into a discipline that uses mathematical thinking, and there are certainly many other ways to learn about logic that do not involve the study of plane geometry.

I would classify mathematical thinking as a form of problem solving. It involves the analysis of given information and the synthesis of the information to discover new facts. This type of reasoning is very difficult. I know of no better way of introducing it to students than through the use of computers. In addition, computer programs must be very logically constructed. *They are, in fact, a proof of the computability of some result.* It goes without saying that computers demand discipline. One cannot deviate one iota from the prescribed rules of syntax (the computer language) or else one is presented with an error message.

Let me try to illustrate some of these ideas. In algebra we teach students how to solve quadratic equations. I presume that the usual procedure is to show them how to complete a square on sample problems and then to do it in general to derive the standard quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

If your students are like mine, they become glassy-eyed somewhere in the middle of the development and stop listening. They realize that all they need do is to recognize a quadratic equation and to memorize the quadratic formula to solve it.

Now suppose you asked students who have learned the quadratic equation to

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write a program to print out the roots of a quadratic equation. They would have to do analysis of the following sort:

1. Read in a, b, and c.
2. Check whether $a = 0$, because in that case they have a linear equation and can't use the formula.
3. Compute the discriminant, $d = b^2 - 4ac$.
   a) If $d > 0$, they will transfer to a part of the program where real roots are handled.
   b) If $d = 0$, they will transfer to a part of the program where repeated roots are handled.
   c) If $d < 0$, they will transfer to a part of the program where complex roots are handled.
4. Print the results, making it clear what case they are in.
5. Provide the capability to repeat the program on a new set of data.

Compare the mental activity required to solve a quadratic equation using the formula with the mental activity required to program the computer to use the formula. In the first instance the problem can be reduced to rote memorization, as it usually is; and this does not involve the use of logic or problem-solving techniques except in a very meager way. However, the computer program required to solve this problem requires an understanding of the formula, the ability to distinguish all the possible cases that might arise, and the ability to explain to an idiot (the computer) the precise instructions it must follow to obtain the desired results.

As a second example, consider the problem of solving a system of linear equations. Students usually learn to solve systems by substitution or elimination. Because of the large amount of calculation involved, we usually restrict our attention to two- or three-dimensional systems. Substitution works fine on two-dimensional systems, is unwieldy on three-dimensional systems, and is an unmitigated mess for systems of higher order. Elimination is a good method, but is usually taught in a non-algorithmic way. Since we restrict our attention to low-order systems, we attempt to teach our students to recognize from the structure of the problems which variable should be eliminated. This approach is hopelessly inadequate for higher order systems, and since students do not learn the algorithmic approach (essentially Gaussian elimination) they are unable to tackle successfully higher order problems.

Now, if you want to program a computer to solve a system of linear equations, you must have a thorough understanding of the elimination algorithm. You must understand that interchanging the equations does not affect the solution, or that multiplying an equation by a constant does not change the solution. You learn that the variables are of no consequence—it is the coefficients that determine the solution. You have a natural vehicle for introducing matrix algebra and determinants, because you must store the coefficients of the system in a two-dimensional array. Finally, the elimination algorithm leads very naturally to the consideration of special cases such as a row of zeros, except in the last position, or a row of all zeros.

> It requires a tremendous amount of problem-solving ability to make a computer solve a problem.

Programming the solution of a linear system is certainly not a trivial task and should probably be one of the last topics taught in a year-long course. But the payoff is large. A great deal of analysis and synthesis must be accomplished. In addition, the computer solution leads naturally to a study of matrix algebra.

Of course, there are many other problems one could treat. Finding the roots of a polynomial by the bisection method is an easy problem. Sorting a set of numbers into ascending or descending order is easy to do without the aid of a computer, but it is an intellectual challenge to program a computer to do the sort. It is easy to think of a variety of everyday-life problems that can be programmed. For example, one might write a checkbook balancing program or a simplified income tax computing program.

My point is this: It requires a tremendous amount of problem-solving ability to make a computer solve a problem. It requires the kind of analytical thinking we want our students to learn. In addition, it is immensely practical to learn how to program. In the next ten years, it is likely that most college students will learn how to program computers and will use the knowledge gained (the problem-solving techniques) in their jobs. With the advent of microcomputers such as the TRS-80 and Apple, which are very cheap and have tremendous computing capability, there is a revolution coming in the use of computers. It will not be uncommon for households to have microcomputers. They will be used to play games and maintain household financial records. People with programming skills will have the ability to take full advantage of them. It behooves us to educate our students in their uses and limitations.

Computers provide feedback to the programmer that is unlike the information we give our students. Syntactical errors are highlighted almost instantaneously. There is never any doubt about intentions or meaning. The computer accepts only what it is programmed to accept, and nothing else. Students must learn to follow the rules, and if they don't, the computer provides immediate feedback.

The analysis required to test and debug a program is very valuable. One must understand the structure of a problem in order to provide an adequate set of test data. Trying to decide why the machine did what it did instead of what you wanted it to do is a very worthwhile experience in analytical thinking.

My experience with junior and senior high school students leads me to believe that the computer can serve as a device for motivating students to learn more mathematics. Usually their first exposure to a computer comes when they play a computer game. After the initial challenge of game-playing is over, they often want to learn how to program their own games or how to solve a mathematical problem. With a little direction from a teacher, their mathematical horizons can be broadened appreciably.

2500 years of Euclid might be enough.

In order to accommodate the use of computers, I propose the following curriculum for college-bound students:

Freshman year	Algebra 1
Sophomore year	Algebra 2
Junior year	Computer programming
Senior year	Precalculus survey

Electives might include plane geometry, calculus, or a higher level computer course.

This proposal was not practical just a few years ago because the cost of computers was exorbitant. That is no longer the case. Over a period of a few years any high school can acquire enough microcomputers to handle all student needs.

So let's get going. It might take some courage to drop good old Euclidean geometry, but I think you would like the results; 2500 years of Euclid might be enough.

COMPUTER-ASSISTED INSTRUCTION IS NOT ALWAYS DRILL

Microcomputers can help students understand random samples and other statistical concepts.

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If you say you have computer-assisted instruction in your school, many educators will think of drill work delivered by a computer. Some will think of simulation activities in which students "run city government" or "mix dangerous chemicals." Others who have had computer-programming experiences will think of problem solving. While teaching a statistics class, I discovered still another use of the computer to aid instruction.

Calculators have been used in my introductory course in statistics to facilitate a "number crunching" aspect of the course. Recently, a microcomputer was added to the list of materials supporting the course. Prior to the time the computer assisted in the instruction, I had some difficulty in teaching a particular concept in sampling. Most students understood that generalizations could be made from the sample to the population when the size of the sample almost equaled the size of the population. However, they were skeptical when the sample represented a smaller portion of the population.

Initially, the students examined the information on table 1 and table 2.

Table 1 indicates the means of ten samples of size 10 drawn at random from a population of size 1000. Table 2 indicates the means of ten samples of size 50 drawn from the same population. The students compared the differences in the population mean and standard deviation and the means and standard deviations in the two different size samples. The students found a smaller range of means on table 2 than on table 1. They concluded that the trend of a smaller range of sample means with larger and larger samples would continue. My current statistics class was treated to a similar activity, but with more dramatic results.

TABLE 1
Ten Samples of Size 10 Drawn from a Population of Size 1000

Samples	Means	Standard Deviations
10	16.4	11.2
10	19.9	12.0
10	20.1	15.4
10	20.4	8.8
10	23.7	16.4
10	25.5	10.7
10	26.5	17.6
10	27.1	14.0
10	27.3	14.6
10	29.0	15.6
Population	25.5	14.4

Instead of distributing the two tables to the students, the class was allowed to choose its own small sample size and large sample size. Summary sheets similar to the tables were generated on the microcomputer and displayed on the screen (see table 1 in the Appendix for the program). This live demonstration sparked the students to begin speculating on the effects of

TABLE 2
Ten Samples of Size 50 Drawn from a Population of Size 1000

Samples	Means	Standard Deviations
50	21.8	14.8
50	22.5	15.0
50	22.8	13.5
50	26.1	14.7
50	26.3	15.5
50	26.8	13.5
50	27.4	14.9
50	27.5	13.0
50	27.5	14.2
50	28.2	15.3
Population	25.5	14.4

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selecting even more sample sizes. A discussion of random selection and sample size followed that seemed to involve more students than had ever been involved before. The students selected their own sample sizes and speculated on the range of ple means. Occasionally, a large ple size resulted in an unusually large range of sample means. Initially, students were puzzled at the "error." Then one student remarked, "But. that's just the nature of random numbers!!" This insight helped others to internalize what had happened. I felt most of the benefits of the lesson were over. I was wrong.

Later in the course, we were discussing control groups and experimental groups. Several students remarked that it was possible to select two samples at random from the same population and have the samples be significantly different. They each referred to the computer activity that was previously cited, where samples were selected from the same population at random. The comment was made, "How do you know we didn't accidentally select a large sample and a small sample?" No student had recalled this idea when I used the summary sheets as handouts.

Perhaps computers can assist instruction by developing mathematical concepts that are remembered better through a live demonstration. Other live demonstrations have been used where students predicted what would happen and then let the computer verify (or refute) their predictions.

Usually the development of linear regression has included a brief comment about other predictive procedures. The students understood that the line that best fit a scattergram might not be straight. However, I had no success in developing an intuitive feeling for multiple linear regression. A computer program was used that was successful in developing this intuitive feeling. The program generated three tests A, B, and C and produced correlation coefficients between Tests A and C and Tests B and C. The user was prompted to pool Test A and Test B to form a new test, Test X. The computer indicated a correlation between Test X and Test C. The scores in Test X were formed by the equation

$$X = (\quad)A + (\quad)B.$$

The user supplied the weightings for Test A and Test B. A copy of the computer program appears in table 2 in the Appendix. A sample run with coefficients appears in table 3.

TABLE 3

YOU TRIED $X = (3)A + (4)B$

TEST A	TEST B	TEST X	TEST C
1	2	11	5
2	1	10	8
3	9	45	15
3	6	33	8
6	16	82	12
0.615	0.703	0.692	

IS THERE A BETTER CORRELATION WITH A DIFFERENT WEIGHTING?

After a few trials, all students seemed to know which test to give more weight in the pooling process. I asked if the weighting had to be positive, and immediately several suggestions for negative weightings were tried by the students.

Following the computer activity, the students reported that the test with the higher correlation should be given more weighting. I asked how the process could be extended to Tests A, B, C, and D. A student suggested a formula for generating scores for Test X,

$$X = (\quad)A + (\quad)B + (\quad)C$$

along with the proper order for weightings on Tests A, B, and C.

I was pleased with the results of the introductory activities and quite pleased that the total time for the computer activity and discussions was less than twenty minutes.

Computer programs such as these have helped me to emphasize statistical thinking and de-emphasize the arithmetic of statistics. Perhaps this shift in emphasis has been the most important aspect of using computers in the classroom to aid instruction.

APPENDIX

Note: The TRS-80, Level II BASIC uses several func-

(Continued on pages 691, 715)

TABLE 1

```
1 REM .... JIM HUTCHESON, TRS-80, LEVEL II
2 REM .... PROGRAM GENERATES POPULATION OF 1000 RANDOM #S
3 REM .... EACH 1-50. USER SELECTS SAMPLE SIZES
4 REM .... PROGRAM RETURNS MEAN & SD OF 10 SAMPLES ORDERED
5 REM .... ON MEANS ... PROGRAM RETURNS POPULATION MEAN & SD
10 CLS
20 PRINT"PROGRAM IS LOADING 1000 NUMBERS FOR THE POPULATION"
25 PRINT:PRINT
30 PRINT"A SHORT PAUSE ...(ABOUT A MINUTE)..."
35 S = 0:S1 = 0
40 DIM A%(1000),M(10),S(10)
50 FOR J = 1 TO 1000
55 A%(J) = RND(50)
60 PRINT@475,J,A%(J)
70 S = S + A%(J):S1 = S1 + A%(J)↑2
80 NEXT J
90 PRINT@475,"POPULATION LOADED"
100 FOR J = 1 TO 300:NEXT:CLS
110 PRINT"NOW SELECT A SAMPLE SIZE...10 SAMPLES WILL BE SELECTED"
120 PRINT"FROM THE POPULATION AT RANDOM"
130 PRINT:PRINT"THE MEANS AND STANDARD DEVIATIONS WILL BE GIVEN"
140 PRINT"THE COMPUTER WILL ORDER THE SAMPLES ON THEIR MEANS"
150 PRINT:PRINT"HOW LARGE DO YOU WANT EACH SAMPLE TO BE";
160 INPUT N
170 CLS
180 FOR J = 1 TO 10
190 A = 0:B=0
200 FOR K = 1 TO N
210 X = A%(RND(1000))
220 PRINT@470,"SAMPLE",J,"SELECTED";K
230 A=A+X:B=B+X*X
240 NEXT K
250 M(J) = A/N
260 S(J) = SQR((B-A↑2/N)/(N-1))
270 NEXT J
280 CLS:PRINT"NOW ORDERING ON SAMPLE MEANS"
290 FOR J = 1 TO 9
300 FOR K = J+1 TO 10
310 IF M(J)<=M(K) THEN 350
320 H = M(J):M(J) = M(K):M(K)=H
330 H = S(J):S(J) = S(K):S(K) = H
350 NEXT K
360 NEXT J
370 PRINT:PRINT"SAMPLE SIZE        MEAN        STANDARD DEVIATION"
380 FOR J = 1 TO 10:PRINTN,M(J),S(J):NEXTJ
390 PRINT:PRINT"POPULATION MEAN ";S/1000,SQR((S1-S↑2/1000)/1000)
400 PRINT"PRESS <ENTER> TO GENERATE OTHER SAMPLES";
405 INPUT Y
410 CLS:GOTO 150
```

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TABLE 2

```
10 REM ... JIM HUTCHESON, TRS-80, LEVEL II
20 REM ... MULTIPLE LINEAR REGRESSION
30 REM ... USER ENTERS WEIGHTINGS - COMPUTER SHOWS PEARSON'S R
40 REM ... USER TRIES TO GET A BETTER R WITH BETTER WEIGHTINGS
50 REM ... PROMPT OF NEGATIVE WEIGHTINGS IS NEEDED
60 REM
70 REM
80 DIM A(5),B(5),C(5),T(5)
90 CLS:PRINT"POOLING SCORES IN A MULTIPLE LINEAR REGRESSION-TEST"
100 PRINT:PRINT"TESTS ARE A,B,AND C"
110 PRINT"SAMPLE NUMBERS ARE ALREADY LOADED"
120 S1=0:S2=0:S4=0:S5=0:S6=0:S7=0:S8=0
130 FOR J = 1 TO 5
140 READ A,B,C
150 A(J)=A:B(J)=B:C(J)=C
160 S1=S1+A:S2=S2+A*A
170 S3=S3+B:S4=S4+B*B
180 S5=S5+C:S6=S6+C*C
190 S7=S7+A*C:S8=S8+B*C
200 NEXT J
210 DATA 1,2,5,2,1,8,3,9,15,3,6,8,6,16,12
220 PRINT"PEARSON'S R FOR TEST A AND TEST C"
230 R1 = (S7-S1*S5/5)/SQR((S2-S1↑2/5)*(S6-S5↑2/5))
240 R1=INT(1000*R1+.5)/1000
250 PRINT" R1 = ";R1
260 PRINT:PRINT"PEARSON'S R FOR TEST B AND TEST C"
270 R2 =(S8-S3*S5/5)/SQR((S4-S3↑2/5)*(S6-S5↑2/5))
280 R2=INT(R2*1000+.5)/1000
290 PRINT" R2 = ";R2
300 PRINT:PRINT"SUPPOSE YOU FORMED A NEW SCORE, SAY X, BY"
310 PRINT"POOLING THE SCORES IN TEST A WITH THE SCORES IN TEST B"
320 PRINT"PRESS <ENTER> TO CONTINUE";:INPUT J
330 CLS:PRINT"TEST A AND TEST C      TEST B AND TEST C"
340 PRINTTAB(1),R1,TAB(24),R2
350 PRINT:PRINT"TEST X WILL BE FORMED LIKE:  X = (3)A + (4)B
360 PRINT"WHAT WEIGHTING SHOULD BE         ↑        ↑"
370 PRINTTAB(33),"HERE    HERE"
380 PRINT:PRINT"LOOK AT THE TWO CORRELATION COEFFICIENTS AND"
390 PRINT"SUGGEST A WEIGHTING. THE COMPUTER WILL FORM TEST X"
400 PRINT"AND COMPUTE PEARSON'S R FOR TEST X AND TEST C."
410 PRINT:PRINT"PRESS <ENTER> TO CONTINUE";INPUT J
420 CLS:PRINT"R1                    R2"
430 PRINT R1,R2
440 PRINT:INPUT"ENTER WEIGHING FOR TEST A",W1
450 INPUT"ENTER WEIGHING FOR TEST B",W2
460 C1=0:C2=0:C3=0
470 FOR J=1TO5
480 T=W1*A(J)+W2*B(J)
490 T(J)=T
500 C1=C1+T:C2=C2+T*T:C3=C3+T*C(J):NEXTJ
510 CLS
520 R3=(C3-S5*C1/5)/SQR((C2-C1↑2/5)*(S6-S5↑2/5))
530 PRINT"YOU TRIED X = (";W1,")A + (";W2,")B":PRINT
540 PRINT"TEST A",TAB(16),"TESTB";TAB(32),"TEST X";TAB(48);"TEST C"
550 FOR J=1TO5:PRINTA(J),B(J),T(J),C(J):NEXTJ
560 R3=INT(R3*1000+.5)/1000
570 PRINT:PRINTR1,R2,R3
580 PRINT:PRINT"IS THERE A BETTER CORRELATION WITH A DIFFERENT WEIGHTING?"
590 GOTO 440
```

COMPUTER-ASSISTED INSTRUCTION IS NOT ALWAYS DRILL

(Continued from page 690)

tions that are different from other versions of BASIC.

1. CLS–clears the screen
2. RND(X)–returns a random number 1 to X if X is greater than 1
3. PRINT@–prints at a position on the screen. The positions are 0–1023

The programs are written in an "endless loop" style. The user is required to depress the BREAK key. This style was chosen over the more conventional END statement or the escape to END question for two reasons:

1. The END statement required the author to anticipate the number of attempts a particular user would need. This varied too often to be practical.
2. The escape question would normally appear at the end of the user's sample selection or weighting. The students in the statistics class made too many typing errors to risk an additional user input. Further, the escape question tends to break the train of thought when the students are concentrating on the output of the computer.

Section VIII

COMPUTER LITERACY

COMPUTER LITERACY—WHAT IS IT?

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The May 1978 *Mathematics Teacher* includes an important position statement regarding "Computers in the Classroom" (prepared by the Instructional Affairs Committee and approved by the NCTM Board of Directors) which states that

> an essential outcome of contemporary education is computer literacy. Every student should have first-hand experiences with both the capabilities and the limitations of computers through contemporary applications. Although the study of computers is intrinsically valuable. educators should also develop an awareness of the advantages of computers both in interdisciplinary problem solving and as an instructional aid.

Whereas such a statement reflects a commendable concern for implementing something called computer literacy, it provides little guidance in explicating what this area of study should include. The National Council of Supervisors of Mathematics, in their 1977 position paper on basic mathematical skills (Feb. 1978 *MT*), do an excellent job of describing the many important components of school mathematics. Their statements of rationale pay particular attention to the fact that "the availability of computers and calculators demand a redefining of the priorities for basic mathematical skills." They go on to include computer literacy as one of ten basic skill areas:

> It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry and business demands an awareness of computer uses and limitations.

It is clear that the mathematics education profession is advocating that some sort of computer literacy experience should be provided for all pupils. (Note that many others, outside the field of mathematics education, are also calling for action in this area: e.g., see the paper by Molnar [1978], which speaks to the importance of education regarding computers and their impact.) However, we need to go beyond these general statements and be more explicit in our statements of possible components (experiences) and desired outcomes (objectives).

The National Science Foundation, through its Research in Science Education program, has awarded a grant to the Special Projects Division of the Minnesota Educational Computing Consortium (MECC) to explore the impact of precollege educational programs designed to increase computer literacy. and the effects of human-computer interaction within the context of instructional computing environments. The MECC Computer Literacy Study (May 1979 *MT*) is concerned with such questions as the following: (1) What is the impact on student knowledge, attitudes, and skills of various approaches to the development of

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For information on two computer literacy programs, see "New Programs" in this issue.—Ed.

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computer literacy? and (2) What impact does using a computer as part of the instructional process in science education have on student attitudes and beliefs about computers? An important initial phase of the study was to search the available literature (articles, curriculum materials, tests, and so on) and attempt to pull together the various views espoused, explicit and implicit, into some coherent set of statements that reflects what is implied by the phrase computer awareness or computer literacy.

If one peruses the literature in this area, it soon becomes apparent that different authors place priorities on very different aspects of computing. On one extreme we find the emphasis placed on knowledge and concepts related to hardware and programming, and at the other the emphasis is on an awareness of applications and issues. This is not to say that the literature can be easily divided into two disjoint sets, as most authors espouse some combination of these ideas. Whereas most authors (including those developing course outlines at the school level) identify the material to be included, explicitly or implicitly, they generally indicate their recommendations for exclusion through omission. Thus it is often difficult to ascertain whether there is really a debate regarding topics or if the curriculum recommendations merely reflect interests and local concerns. However, since the views appear to be so diverse, the project elected to develop a conceptual framework that attempted to incorporate as much as possible the views of all. That is, the resulting description should describe computer literacy or awareness in its broadest sense. This flexible, general approach is a somewhat less absolute view, and it was felt that such an approach would enable us to assess student knowledge and attitudes across a wide range of possible topics as well as focus on particular aspects of hardware, software, applications, and issues. In addition, such an approach should provide a basis for further discussion regarding the priority that might be given to particular aspects of computing.

As a first step, information on a wide range of courses was collected (in the United States and in Great Britain) through personal contacts, responses to requests for information that appeared in various journals, and communications with the institutions cited in the "HumRRO exemplary institutions" book (Human Resources Research Organization, 1977). (More complete information on courses and materials is available in the final report of the project.) One indication of the extensive nature of this "collection phase" is the fact that responses in the form of course descriptions, course objectives, curriculum guides, or evaluation instruments were received from about fifty school districts throughout the country. Whereas many of these were similar, there were sufficient differences (both with the outlines and other text sources) to justify the earlier decision to attempt to explicate the concept in a broad sense and *not* attempt to survey common practice or report on the set of common recommendations.

In addition to course descriptions, nearly 2000 test items relating to computers were collected, judged for quality, and categorized. The course descriptions and test items were then used to develop a rather comprehensive list of the topics and objectives covered in the courses. These were grouped under six main headings: Hardware (H): Programming and Algorithms (P); Software and Data Processing (S): Applications (A): Impact (I): and Attitudes, Values, and Motivation (V). The first five headings deal with cognitive outcomes and the sixth is concerned with attitudes, or the affective domain. The list of objectives was subjected to many revisions and then circulated to twelve "outside experts" representing the professional societies, industry, and education, for reactions and validation. On the basis of feedback from these individuals, the list was revised once more and the current version is presented in this paper.

Before moving to the actual statements it is important to provide some information about format of the lists. The objectives are really informational objectives. Whereas some are stated rather specifically, explicitly designating a desired outcome, for the most part they are not behavioral but represent guides for the construction of test items. Since subsequent research will involve groups of students in schools, there was also a need to try to reduce the set to some smaller subset that could be assessed in a reasonable period of time. Thus the listing contains a number of starred statements; these are referred to as "core objectives" and as such represent the set to be used in the research. In general, the outside reviewers suggested that, whereas the core was appropriate for the research task, there is a need to extend this set, as most of the statements so designated are only representative of the lower levels of cognitive skills and understandings. Thus the reader should not attach the idea of "minimum competency" to this core set, but rather recognize that this is only minimal for describing what might be called computer awareness under the condition that the subsequent assessment would be manageable within the constraints imposed by the large-scale study.

Computer Literacy Objectives—Cognitive

Hardware (H)[a]

*H.1.1 *Identify* the five major components of a computer: input equipment, memory unit, control unit, arithmetic unit, output equipment.

*H.1.2 *Identify* the basic operation of a computer *system:* input of data or information, processing of data or information, output of data or information.

*H.1.3 *Distinguish* between hardware and software.

*H.1.4 *Identify* how a person can access a computer: for example,

1. via a keyboard terminal
   a. at site of computer
   b. at any distance via telephone lines
2. via punched or marked cards
3. via other magnetic media (tape, diskette)

*H.1.5 *Recognize* the rapid growth of computer hardware since the 1940s.

*H.2.1 *Determine* that the basic components function as an *interconnected system* under the control of a *stored program* developed *by a person.*

*H.2.2 *Compare* computer processing and storage capabilities to the human brain, listing some general similarities and differences.

Programming and Algorithms (P)

Note: The student should be able to accomplish objectives 1.2–2.5 when the algorithm is expressed as a set of English language instructions and is in the form of a computer program.

P.1. *Recognize* the definition of "algorithm."

*P.1.2 *Follow* and give the correct output for a simple algorithm.

*P.1.3 Given a simple algorithm, *explain* what it accomplishes (i.e., interpret and generalize).

*P.2.1 *Modify* a simple algorithm to accomplish a new, but related, task.

P.2.2 *Detect* logic errors in an algorithm.

P.2.3 *Correct* errors in an improperly functioning algorithm.

P.2.4 *Develop* an algorithm for solving a specific problem.

P.2.5 *Develop* an algorithm that can be used to solve a set of similar problems.

Software and Data Processing (S)

S.1.1 *Identify* the fact that we communi-

* Denotes core objectives.

a. Note that the coding is H—Hardware, P—Programming and Algorithms, S—Software and Data Processing, A—Applications, and I—Impact. Also, for each statement the first digit after the letter refers to a cognitive level—1 indicating a low level, generally a skill or knowledge of facts, and 2 standing for a higher level of understanding, requiring some analysis and/or synthesis. The final digit is merely a count of items within each level. Whereas no priority is intended with the final digit, there has been an attempt to place the ideas in some sort of logical sequence.

cate with computers through a binary code.

S.1.2 *Identify* the need for data to be organized if it is to be useful.

S.1.3 *Identify* the fact that information is data that has been given meaning.

S.1.4 *Identify* the fact that data is a coded mechanism for communication.

S.1.5 *Identify* the fact that communication is the transmission of information via coded messages.

*S.1.6 *Identify* the fact that data processing involves the transformation of data by means of a set of predefined rules.

*S.1.7 *Recognize* that a computer needs instructions to operate.

*S.1.8 *Recognize* that a computer gets instructions from a program written in a programming language.

*S.1.9 *Recognize* that a computer is capable of storing a program and data.

*S.1.10 *Recognize* that computers process data by searching, sorting, deleting, updating, summarizing, moving, and so on.

*S.2.1 *Select* an appropriate attribute for ordering of data for a particular task.

S.2.2 *Design* an elementary data structure for a given application (that is, provide order for the data).

S.2.3 *Design* an elementary coding system for a given application.

Applications (A)

*A.1.1 *Recognize* specific uses of computers in some of the following fields:

a. medicine
b. law enforcement
c. education
d. engineering
e. business
f. transportation
g. military defense systems
h. weather prediction
i. recreation
j. government
k. the library
l. creative arts

A.1.2 *Identify* the fact that there are many programming languages suitable for a particular application for business or science.

*A.1.3 *Recognize* that the following activities are among the major types of applications of the computer:

a. information storage and retrieval
b. simulation and modeling
c. process control—decision-making
d. computation
e. data processing

*A.1.4 *Recognize* that computers are generally good at information-processing tasks that benefit from the following:

a. speed
b. accuracy
c. repetition

*A.1.5 *Recognize* that some limiting considerations for using computers are as follows:

a. cost
b. software availability
c. storage capacity

*A.1.6 *Recognize* the basic features of a computerized information system.

*A.2.1 *Determine* how computers can assist the consumer.

*A.2.2 *Determine* how computers can assist in a decision-making process.

A.2.3 *Assess* the feasibility of potential applications.

A.2.4 *Develop* a new application.

Impact (I)

*I.1.1 *Distinguish* among the following careers:

a. keypuncher/keyoperator

b. computer operator
c. computer programmer
d. systems analyst
e. computer scientist

*I.1.2 *Recognize* that computers are used to commit a wide variety of serious crimes, especially stealing money and stealing information.

*I.1.3 *Recognize* that identification codes (numbers) and passwords are a primary means for restricting the use of computer systems, computer programs, and data files.

I.1.4 *Recognize* that procedures for detecting computer-based crimes are not well developed.

*I.1.5 *Identify* some advantages or disadvantages of a data base containing personal information on a large number of people (e.g., the list might include value for research and potential for privacy invasion).

I.1.6 *Recognize* several regulatory procedures; for example, privilege to review one's own file and restrictions on the use of universal personal identifiers that help to insure the integrity of personal data files.

*I.1.7 *Recognize* that most "privacy problems" are characteristic of large information files whether or not they are computerized.

*I.1.8 *Recognize* that computerization both increases and decreases employment.

*I.1.9 *Recognize* that computerization both personalizes and impersonalizes procedures in fields such as education.

*I.1.10 *Recognize* that computerization can lead to both greater independence and dependence on one's tools.

*I.1.11 *Recognize* that, whereas computers do not have the mental capacity that humans do, through techniques such as artificial intelligence, computers have been able to modify their own instruction set and do many of the information-processing tasks that humans do.

*I.1.12 *Recognize* that alleged "computer mistakes" are usually mistakes made by people.

*I.2.1 *Plan* a strategy for tracing and correcting a computer-related error, such as a billing error.

I.2.2 *Explain* how computers make public surveillance more feasible.

*I.2.3 *Recognize* that even though a person does not go near a computer, he or she is affected indirectly because the society is different in many sectors as a consequence of computerization.

I.2.4 *Explain* how computers can be used to effect the distribution and use of economic and political power.

Computer Literacy Objectives—Affective

Attitude, Values, and Motivation (V)[b]

*V.1 *Does not feel* fear, anxiety, or intimidation from computer experiences.

*V.2 *Feels* confident about his or her ability to use and control computers.

*V.3 *Values* efficient information processing provided that it does not neglect accuracy, the protection of individual rights, and social needs.

*V.4 *Values* computerization of routine tasks so long as it frees people to engage in other activities and is not done as an end in itself.

*V.5 *Values* increased communication and availability of information made possible through computer use provided that it does not violate personal rights to privacy and accuracy of personal data.

V.6 *Values* economic benefits of computerization for a society.

b. The coding scheme, V.1–V.9, is merely for recording purposes and is not intended to convey any priorities or hierarchy.

V.7 *Enjoys and desires* work or play with computers, especially computer-assisted learning.

V.8 *Describes* past experiences with computers with positive-affect words, like fun, exciting, challenging, and so on.

V.9 Given an opportunity, *spends* some free time using a computer.

Summary

The lists of objectives in the previous section could use further description. In particular, note that A.2.1 and A.2.2 speak to the notion of assisting the consumer or in decision making. In their phrasing, these objectives tend to suggest the positive aspects of computing. Of course, they must also include aspects of the limitations of the machine, in particular those aspects of decision making that relate to ethical and moral considerations.

One further point deserves mention here. Whereas the objectives are intended to provide a broad perspective, there is one area that has been omitted. This is the detail of early history (e.g., Babbage and Hollerith). This was a conscious decision based on discussions among the investigators and input from the review panel and other knowledgeable persons. This is not to suggest that the topic is not interesting or motivating, but rather that it reflects the bias that such knowledge is not really functional or prerequisite to an understanding of the other areas.

The lists of objectives in the previous sections are quite extensive and, of course, complete coverage is probably not feasible within any single course and even less feasible as an addition to the school mathematics curriculum. However, if we view computer literacy in this broad sense, it is now appropriate to "pick and choose," if you will, and allocate selected activities/experiences to various points in the total school curriculum, including mathematics. It may also be desirable to allocate a portion of some year to a course called computer awareness or computer literacy and include those objectives that require some reasonable time for study. Such decision making is a job for those most suited for the task—the teachers in the school.

Whereas these lists should provide a basis for such decisions, much more remains to be done. In particular, it should be noted that the language used in many of the statements will probably need to be rephrased if it is to be communicated to younger students. Also, the objectives will need to be translated into a form appropriate for assessment. The research project has developed items for those statements designated as core and will disseminate information about the tests in the near future. Of course, the assessment is most appropriate for determining the effectiveness of a given computer literacy curriculum, and at present, this is a relatively undeveloped area.

Reaction to the ojectives is welcome; the statements are all open to revision and you may feel we have omitted some important points. The project has also produced other reports including further descriptive information on the research; the results of a statewide (Minnesota) survey of all secondary school science, mathematics, and business teachers (as well as selected teachers in other disciplines); and summaries of selected references. Questions should be directed to Computer Literacy Study, Minnesota Educational Computing Consortium, 2520 Broadway Drive, St. Paul, MN 55113.

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COMPUTER LITERACY—WHAT SHOULD IT BE?

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The way things are is not necessarily the way they should be.

The February 1980 *Mathematics Teacher* contains a significant report of research conducted for the National Science Foundation by Johnson et al. (1979) at the Minnesota Educational Computing Consortium (MECC). The report, titled "Computer Literacy—What Is It?" has already generated a great deal of discussion among teachers and others attempting to understand the place of computing in the precollege curriculum. At the 1980 annual meeting of the National Council of Teachers of Mathematics one of the best attended sessions was a report on this project by one of the MECC authors (Klassen).

The goal of defining *computer literacy* is of great importance. Much hangs on it. If the public decides that the subject is worth teaching in its schools, then it faces an equipment bill of about $1 billion in the United States alone. It faces the additional costs of curriculum development, of teacher training, and of the assessment of student achievement. It is obvious that the way computer literacy is *defined* will have a profound effect on the public's willingness to *support* the teaching of computing in their schools.

The specific concern that prompts the present article is that many readers of the February *MT* report are making the mistake of seizing upon the list of sixty-three items (which occupies half of the report and is given the boldface title "Computer Literacy Objectives") and interpreting the list as providing an objective measure of what computer literacy *should* be.

Whereas the organization and physical appearance of the *MT* article may invite such an interpretation, and, indeed, the authors may wish us to do so, they also explain clearly that their methodology in drawing up the list results in little more than an empirically gathered collection of the educational objectives actually found in existing computer courses in 1978–79. Thus, readers should not conclude that there is any moral imperative to teach those things. In the latter half of this communication I will go a step farther than the MECC authors. *I will argue that fully four-fifths of these empirically discovered objectives should not be used in any significant definition of computer literacy.* More of that later.

This observed tendency to interpret the MECC list of objectives as authoritative, imperative, or official is aggravated by the publication and dissemination of an impressive 1979 MECC document bearing the title "Minnesota Computer Literacy and Awareness Assessment," and a credit line indicating NSF grant support. Who could blame a casual reader for inferring that these fifty-three multiple-choice content questions provide an authoritative, official definition of what a course in computing *should* be aimed at? "At last," one can almost hear the harried teacher say, "now I can see whether I'm teaching the right things." *In fact, as I will soon argue, the teacher who is teaching toward that test instrument is teaching the wrong things.*

Equipment could cost $1 billion in the United States.

I do not take the MECC authors to task

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for this public confusion about the results of their project, but only call attention to the fact that confusion does exist about what a course in computing *should* be. The MECC test instrument was developed purposely to find out what students are learning in *today's* computer literacy courses. It is very good for such empirical work and certainly should not be interpreted as setting a standard or a goal. Furthermore, the authors of the *MT* article precede the list of objectives with this extremely forthright caveat:

> Most of the statements [designated as core objectives] are only representative of the lower levels of cognitive skills and understandings. Thus the reader should not attach the idea of "minimum competency" to this core set, but rather recognize that this is only minimal for describing what might be called computer awareness... (p. 93)

What Is Wrong with the MECC Objectives?

Little or nothing, if we use them to define *computer awareness*, as the authors warn us that we should. Everything, if they are to define *computer literacy*.

The argument proceeds by analogy. Literacy in a language means the ability to read and write, that is, to *do* something with language, not merely to *recognize* that language is composed of words, to *identify* a letter of the alphabet, or to be *aware* of the pervasive role of language in society. Literacy in mathematics means the ability to add numbers, solve equations, and so on—to *do* mathematics, not merely to *recognize* that numbers are written as sets of digits or to *identify* a fraction by its appearance or to be *aware* of the vocational advantages of being able to do mathematics.

By analogy, computer literacy must also mean the ability to *do* computing, and not merely to recognize, identify, or be aware of alleged facts about computing. The basic flaw in attempting to apply the MECC objectives as a standard for what *should* be taught in a computer literacy course is that of the sixty-three objectives given, only twelve require that the student be able to do anything. (Significantly, eight of the nine objectives in the "Programming and Algorithms" section fall into this category along with three of the thirteen in "Software and Data Processing.") The other fifty-one objectives involve nothing more than student acceptance of *hearsay knowledge* about computing, such as might be acquired from a book or from being told by the teacher.

This category of knowledge, the lowest in Plato's hierarchy, is essentially verbal. Its acquisition involves the student mainly in encoding words and remembering them when an appropriate stimulus is presented. It is qualitatively different from the knowledge that comes from experience: *doing* writing, *doing* mathematics, or *doing* computing.

Consider the following objective (H.1.1 in the MECC list): "*Identify* the five major components of a computer: input equipment, memory unit, control unit, arithmetic unit, output equipment" (Johnson et al., p. 93). This "core" objective is tested for by item 68 in the MECC assessment instrument: "In addition to input and output equipment, computers contain": for which the correct response is "memory units, control units, arithmetic units" (MECC 1979). Clearly, the student who has read and memorized the classical definition of a computer will ace this item.

> Computer literacy is the ability to do computing.

Yet we should all be worried by the fact that months and years can go by in the life of a professional computer scientist without any need to remember or make use of or even reflect on the fact that somewhere in the bowels of a computer lies an arithmetic unit and somewhere else (a few microns away on the same chip, nowadays) lies a control unit, and that they are, logically at least, distinct. He or she could create an entire management information system or an airline reservations system or a numerical analysis package without ever calling on that piece of knowledge or putting it to use.

Except for a few who work at or near the hardware level, people who *do* computing rarely exercise that bit of textbook knowledge.

An even greater cause of worry about the misuse of objectives like this one comes out of direct experience in working with thousands of children and adults who are actively gaining firsthand experience in using computers. After only a few hours of such laboratory experience, they know enough to score near the top on the handful of test items based on the dozen MECC objectives requiring that the student be able to *do* computing. Yet these same students do not have the foggiest idea about what an arithmetic unit is or a control unit is or what the difference is. (They do know about *input*, *output*, and *processing*, by the way, because they experience these things and have a need for words to describe them.)

Performance objectives are needed.

Yet these doers of computing have a knowledge qualitatively superior to that of the hearers about computing. The Chinese proverb says it well: "I hear, and I forget. I see, and I remember. I do, and I understand." The doers should not be punished by misapplication of tests and objectives derived from classes where hearsay knowledge is the principal commodity.

I have criticized one objective at length, but the same criticism applies to four-fifths of the rest in the MECC list. Here are a few representative ones, taken only from the starred "core objectives" in the February 1980 *MT* article:

H.1.5 *Recognize* the rapid growth of computer hardware since the 1940s.

S.1.6 *Identify* the fact that data processing involves the transformation of data by means of a set of predefined rules.

A.1.4 *Recognize* that computers are generally good at information-processing tasks that benefit from the following: a) speed, b) accuracy, c) repetition.

A.2.1 *Determine* how computers can assist the consumer.

I.1.8 *Recognize* that computerization both increases and decreases employment.

Altogether there are eleven objectives that start with "Identify," twenty-one that start with "Recognize," and three that start with "Determine." In every such case little real change occurs if these verbs are replaced by the single word *Remember*. Indeed, the corresponding test items on the MECC assessment instrument require only that the student remember the "right" set of words that goes with that objective.

To make clear the contrast between hearsay knowledge and knowledge that comes out of practice, consider the very different flavor of the following sampling of the twelve MECC objectives *not* in the hearsay category:

P.1.2 *Follow* and give correct output for a simple algorithm.

P.2.1 *Modify* a simple algorithm to accomplish a new, but related, task.

P.2.4 *Develop* an algorithm for solving a specific problem.

S.2.2 *Design* an elementary data structure for a given application.

Any course in computer literacy ought to spend four-fifths or more of its time and effort on *performance* objectives, such as the ones in this latter group. Going further, I would argue that it is intellectually improper to inculcate beliefs and values about a subject that do not arise out of the student's direct experience with the content of that subject. If I were writing about mathematics or reading and writing, there would be little disagreement about this point. Readers of this journal, for example, would be properly outraged if they were asked to spend four out of five days working on Johnny's and Janey's beliefs and values about the subject of mathematics and to spend the other day teaching them to do mathematics. However much we want our students to remember facts about, and feel good about, mathematics, we know that these beliefs and values will be short lived if our students go out into the world with poor ability to do mathematics.

The Nub of the Problem

When one remembers the goals and the methodology of the MECC study, it is easy to see how that particular list of objectives came into being. The study's goal was to assess "the impact on student knowledge, attitudes, and skills of various approaches to computer literacy." The authors tell us that they defined computer literacy as broadly as possible, so as not to leave out any approach. Their methodology, then, was to collect "course descriptions, course objectives, curriculum guides (and) evaluation instruments" from *existing* courses of study in various school districts. The resulting list of objectives is a somewhat cleaned-up union set of all the stated and implied objectives of actual courses then in place (1978–79).

That constraining phrase, "then in place," is the nub of the problem that arises if one is looking for guidance as to what *should* be taught in a computer literacy course today and in the decade ahead.

The vast majority of precollege computing courses then in place were woefully unprepared to give students significant practical experience. Thirty students in a class often had combined access to a single computer or terminal for an hour a day. In an entire semester each student might experience perhaps only two hours of direct use. No one should be surprised, then, that 80 to 90 percent of the student's time in such a course was spent reading and listening to textbook information about computers and computing. Teachers have to teach something, after all, and the MECC study shows clearly that what they were teaching for the most part was computer *awareness*.

It takes much more equipment to be able to turn out literate users of computing. The first equipment plateau is reached at about eight computers available an hour a day for each thirty-student computer class in the school. This arrangement allows half the students to work in pairs in the computer lab while the other half is receiving instruction in the classroom. Alternate-day rotation between lab and classroom gives each student pair about thirty hours of computer practice. The next plateau occurs at about fifteen computers per school. Logistics are the same as with eight computers, but each student can work alone in the lab.

I am familiar with several schools that have reached the first plateau and one entire school district nearby that is well on the way to reaching the second plateau. Although this situation is attainable and adds less than 1 percent to the operating cost of a school, nevertheless, it is still an unusual one to find today. Declining equipment cost and increasing public awareness of the intellectual and vocational value of being a literate computer user will in due course enable schools to do the job property.

Conclusions

The MECC Computer Literacy Study should not be misinterpreted. It gives us a well-documented snapshot of the state of computer education as actually practiced in 1978–79. What we see in the snapshot is a classroom with a lot of reading and listening, a little seeing, and hardly any doing.

The MECC study gives little guidance, however, as to what the learning objectives of a precollege computer literacy course *should* be. Readers of the study deserve blame if they turn to the published MECC objective list as an ideal or if they use the MECC assessment instrument as a standard test of anything more than elementary computer awareness.

A significant challenge remains, then: to define learner objectives for a course that will turn out literate doers of computing, and then to embody those objectives in a practical curriculum intended for wide adoption.

Although I have well-formed ideas about how to do that, the pages of this journal are probably not an appropriate place to present subjective judgments. Suffice it to say that such a curriculum will put primary emphasis on the direct interaction between the computer and the student, with a learner goal of mastering wholly new analytic, expressive, and problem-solving skills.

Computing belongs as a regular school subject for the same reason that reading, writing, and mathematics are there. Each one gives the student a basic intellectual tool with wide areas of application. Each one gives the student a distinctive means of thinking about and representing a problem, of writing his or her thoughts down, of studying and criticizing the thoughts of others, and of rethinking and revising ideas, whether they are embodied in a paragraph of English, a set of mathematical equations, or a computer program. Students need practice and instruction in all these basic modes of expressing and communicating ideas. Mere awareness of these modes is not worth the time it takes away from teaching the creative and disciplined use of these fundamental intellectual tools.

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IN DEFENSE OF A COMPREHENSIVE VIEW OF COMPUTER LITERACY—A REPLY TO LUEHRMANN

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In Arthur Luehrmann's critique of our February 1980 *Mathematics Teacher* article on computer literacy objectives, he proposes a rather narrow view of literacy. Despite his claim, there are two (not just one) generally accepted definitions of *literacy.* One is, as he points out, the ability to communicate, for example, reading and writing; and the other, which he neglects, is the state of being informed, "cultured," and well versed. Whereas the first is a subset of the second, both definitions are commonly used. It is not surprising that the term *computer literacy* shares the semantic ambiguity of language literacy. The narrow view is that computer literacy is simply a matter of doing things with a computer. The comprehensive view is that computer literacy is an understanding of computers that enables one to evaluate computer applications as well as to do things with them.

The comprehensive view of computer literacy is consistent with the long-established tradition of scientific literacy and related formulations such as technological literacy, geographic literacy, and economic literacy, to name, only a few. Scientific literacy is generally defined as the knowledge about science that the layperson needs to function effectively. Scientific literacy refers not only to learning scientific facts but also to one's understanding of the implications of science and science-society issues. Thus it is not surprising that we often see computer literacy equated with "computers and society" and courses on the social role of computers.

The comprehensive view of computer literacy is also consistent with the current recommendations of the National Council of Teachers of Mathematics (1980). In *An Agenda for Action*, they recommend that "a computer literacy course, familiarizing the student with the role and impact of the computer, should be a part of the general education of every student." This view of computer literacy is also in accord with the literature on the conceptualization of computer literacy. For example, Moursund (1976), Rawitsch (1978), and Watt (1980) all define computer literacy in a broad, comprehensive fashion.

Luehrmann's attempt to define computer literacy as simply "doing computing" and his attempt to belittle knowledge of computer systems by calling it "hearsay knowledge" neglect the semantic ancestry of the term and close the door on a broader understanding of computer technology. There are very good reasons for people to be able both to communicate with computers and to be knowledgeable about them.

In defining computer literacy it is useful to distinguish it from computer science. The most succinct distinction is to say that computer literacy is that part of computer

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science that everyone should know or be able to do. Both language literacy and scientific literacy are commonly defined in terms of the layperson and his or her needs. Likewise, computer literacy should be thought of as the knowledge and skills the average citizen needs to know (or do) about computers. This obviously implies that students should be taught more than simply how to operate or program a machine. They also need to know how computers can be productively used and what the consequences of computerization are. Thus matters of computer literacy should be taught not only in the mathematics department but in science and social studies courses as well.

Ordinary people, old and young alike, have very real, practical needs for computer understanding. For example, people need to know enough about computer systems so as not to be intimidated by a computerized billing error; people need to know whether to acquire computer equipment for home or work; people need to learn how to evaluate when computer applications are helpful and when they are harmful; and so forth. Some of these things can be learned as a byproduct of learning to write simple BASIC programs, but most of this type of useful knowledge cannot be learned that way. Indeed we would argue that most of what every ordinary citizen needs to know about computers will not be learned from learning how to program.

Instructional Objectives

In evaluating the instructional objectives we proposed for computer literacy, Luehrmann claims that they are merely a description of what was taught two years ago. In fact, whereas we do not claim that they represent a perfect set, our list of objectives is much more. The list is an evolving conceptual structure and smorgasbord for computer literacy. The objectives obviously deserve ongoing refinement and updating. In the absence of any alternative sets of objectives, we constructed a list that would guide us in the construction of test items. This accounts for the omission of "doing" or computer usage objectives. More recently these objectives have been revised to take into account changing technology and broader concerns. The revised objectives, which appear in Anderson and Klassen (1981), are designed to serve as a conceptual framework for curriculum planning and development. Whereas we were originally constrained by what might be measured with paper-and-pencil tests, recent efforts allow us to add behavioral or psychomotor activities that are central to learning about computers but are hard to measure except by observation.

Objectives are generally specified at one of three different levels: (1) broad learning goals, (2) informational objectives or desired learning outcomes, or (3) behavioral objectives for a specific activity. We have aimed our objectives at the second level, the informational level, but many of these objectives can be used as behavioral objectives as well. Because we intended that the objectives would be worthwhile guides for curriculum materials, we used behavioral language including such terms as *recognize* and *identify*. Whereas some of the objectives deal with factual information that requires the exercise of recall, many of these objectives require a thorough understanding of concepts and principles. In most instances it would be much more appropriate to substitute the word *understanding* for *recognize*. Consequently, Luerhmann's claim that most of the objectives are trivial is totally unfounded.

Furthermore, we know from both our Minnesota assessment of computer literacy (Anderson, Krohn, and Sandman, 1980) and the National Assessment (Carpenter et al. 1980) that a great many junior and senior high school students have basic misconceptions about computers. Even among those who have taken computer programming classes we find a great deal of misinformation. Luerhmann suggests that our objectives and associated test items are of such a low level that after a few hours of hands-on computer experience a typical student would be able to "score near the top." The empirical facts are just the opposite. From the 1978 National Assessment of Mathematics we know that many, if not most, of the students who had taken one semester of computer programming still could not read a simple flowchart (Anderson 1980). In addition, the study, which tested some fifty computer classes (Klassen et al. 1980), found that the average performance on programming items was only 30 percent correct for those students who completed courses where the instructor taught computer programming. A more thorough statistical analysis in this study revealed that the number of hours allocated for hands-on computer activities did not contribute as much to computer learning as other factors such as the type of course and time spent on computer topics (Klassen et al. 1980).

> Computer literacy is not the same as computer science.

Luerhmann proposes that low-level knowledge about computers should be called computer awareness. We would accept such a definition but only if computer literacy is defined to include computer awareness as well as computer programming and other essential ingredients of computer literacy. We believe that computer literacy must encompass the following domains:

programming and algorithms
skills in computer usage
hardware and software principles
major uses and application principles
limitations of computers
personal and social impacts
relevant values and attitudes

From this taxonomy, which is based on our revised list of objectives for computer literacy, it is obvious that we place a significant and strong emphasis on the experiential aspects of computing as well as on those areas traditionally called computer awareness. The comprehensive approach to computer literacy requires that all these domains be included in a curriculum dealing with computer literacy. We do not assume that each domain or each objective will be given equal weight, as Leurhmann seems to assume. We certainly hope that those using our objectives will "pick and choose" from the entire list and will weigh those that they choose.

Negative Aspects of the Narrow View

The narrow view claims that specific types of computer experience and computer programming are the only important components of computer literacy. Those persons who promote this philosophy may unwittingly promote mindless or meaningless "doing" as well as constructive experiences. Without adequate direction, students who are actively "doing computing" may in fact be learning nonrigorous procedural thinking, acquiring misconceptions, and otherwise wasting valuable instruction time. The typical "doing computing" approach consists of teaching students to write a few programs in the BASIC language. As Papert (1980) points out, this learning strategy often results in student learning that stifles creativity and suppresses motivation. It can also lead to awkward or poor algorithmic thinking.

The development of the BASIC language during the 1960s paved the way for the narrow view of computer literacy. BASIC was offered as the single instructional language that everyone was encouraged to learn. The numerous developments in computer science and computer techniques during the 1970s have made this view obsolete. Now there is a wide diversity of computer systems in place, and few of them require the user to write simple BASIC programs. If teachers now focus on using microcomputers to replicate the approach of the 1960s, students will not be fully prepared to deal with the many large, complex systems of the 1980s, either as programmers or as end users.

Another serious problem is that in the brief time of a typical course, the student can learn very little about problem solving and algorithms. It is also difficult to provide experiences with a variety of languages and a variety of computing systems. Students may complete typical instruction with limited and inefficient strategies for

> Hands-on computer experience does not guarantee computer literacy.

problem solving. And if the student only learns about a single computer system, he or she can develop a narrow conception of computing and may have difficulty in transferring those skills to other computer environments.

The solution is not to provide students with computing experiences per se. The solution is to provide students with constructive computer experiences. It is not easy to provide these constructive experiences. The design of instruction is not an easy task. Those that argue for the narrow view of computer literacy should stop telling us what computer literacy should not be and advise us on the specifics of what it should be. We need many minds to work on what contributes to the "ability to do computing" and how we can move toward an ongoing state of computer literacy in schools across the country.

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EXPLAINING COMPUTER RELATED CONCEPTS & TERMINOLOGY

Harold W. Lawson, Jr.

Have you ever tried to explain computer systems to someone who knows nothing or very little about them? Where do you start? Do you first explain the binary number system. Boolean algebra and/or computer logic? Perhaps you begin by showing the classic CPU, memory, I/O block diagram and explain each of these constituents, including stored programs, instruction sets, etc. Maybe you skip all of these low level details and explain flowcharting and/or some relatively simple high level programming language such as Basic.

Regardless of where you begin, you soon recognize that although you may be somewhat successful (after sufficient effort), the person being taught has no idea about computer systems and computing as a whole and many questions remain about the "miracle of the computer." On the other hand, it seems, on the surface, to be difficult to give a complete picture of computer systems right from the beginning.

After several years of experience with a "process oriented" approach we can remove the doubt and say that it is indeed possible (and desirable) to introduce computer systems concepts and terminology right from the beginning in an integrated manner. This approach has resulted in the book *Understanding Computer Systems** and the purpose of this article is to present the basic concepts used in this approach. As with all endeavours of this nature, the success depends upon proper structuring of the presentation. The approach used is highly pictorial, and terminology is introduced bit-by-bit in a logical fashion. Each chapter finishes with a summary, work list and problems. The entire vocabulary introduced is presented at the end in the form of a glossary.

But enough about the structure of the book—let us consider the approach.

Introducing Processes and Systems

"People learn best when new concepts are presented in terms of what they already know."

The process is used as the LCD (lowest common denominator) for explaining the constituents of computer systems. At the beginning, it is necessary to select widely known model processes that can be used as the basis for association with computer system related processes. To show the example used, we take, as a verbatim quotation from the book, the introductory model process descriptions.

**Understanding Computer Systems*, by Harold W. Lawson, Jr., Lawson Publishing Company, Linkoping, Sweden, ISBN 91-7372-333-9. Similar versions of this paper have been previously published in *Computer Age*, issue 13, 1980 under the title "Understanding Computer Systems" and in *Mikrodatorn*, Nr. 2, 1981 under the title "Att Forklara Begrepp Inom Datatekniken." Permission of these publishers has been granted for publication in *Creative Computing*.

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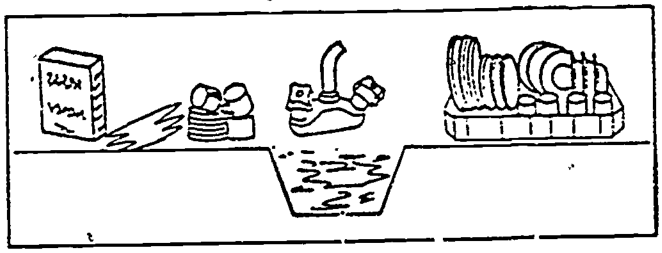
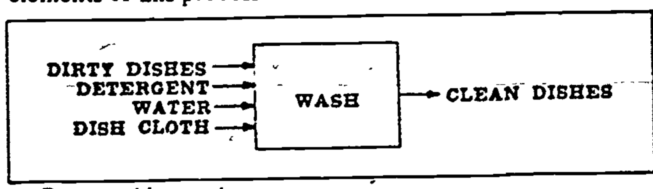
We begin this education process with a pictorial representation of a well-known real life *process* (task) faced by many of us.



A Process.

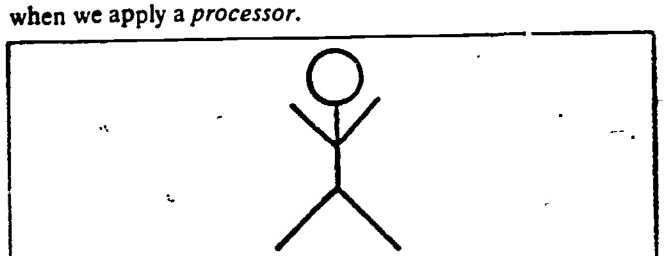
Note that the concept of *process* and *task* are synonymous and thus in all further references to process related concepts, the word "task" may also be used.

Let us now consider this real life process of washing the dishes in the form of an abstraction which shows the major elements of this process as follows:



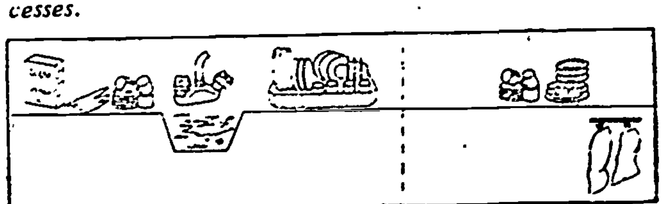
A Process Abstraction.

Dirty dishes, detergent, water and dish cloth are *process inputs* and clean dishes are a *process output*. The process which we have shown here can only be carried out, *executed*, when we apply a *processor*.



A Processor.

Let us complement this single process by introducing a second process, thus creating a *system* of *cooperating processes*.



A Second Process

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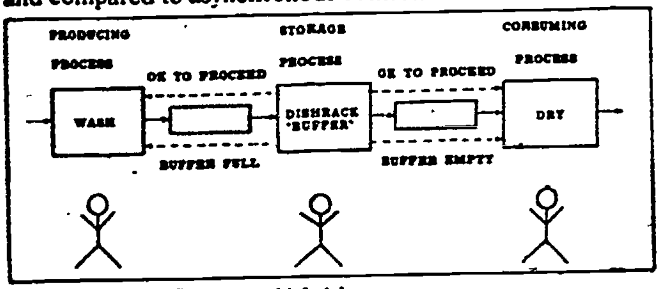
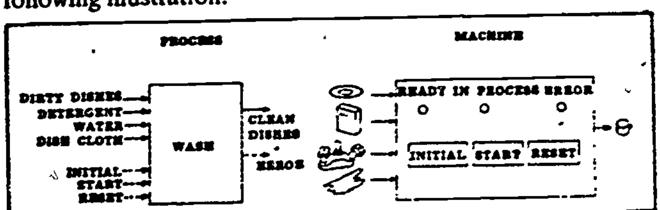
In explaining control, we consider *signals* to processes and related control mechanisms. For example, the following illustrates the producer-consumer relationship of asynchronous control where the dishrack provides a perfect example of a buffer. The concept of synchronous control is then explained and compared to asynchronous control.



Asynchronous Control of Multi-processors

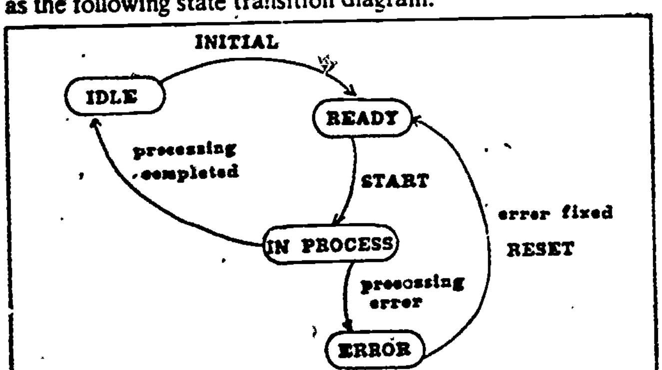
In finalizing the control notions, an important analogy is made by presenting the process as a machine with lights (corresponding to states) and pushbuttons as illustrated in the following illustration:



A Process as a Machine Controlled by Push-buttons

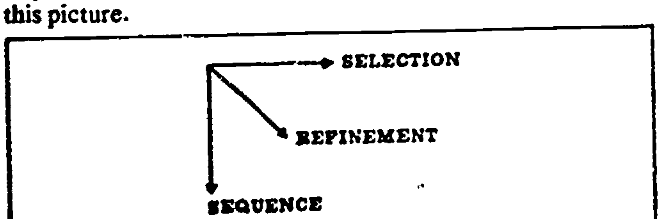
Programming Concepts

After considering the WASH process as a pushbutton machine, it is a simple matter to present the logic of a process as the following state transition diagram.



State Transition Diagram

To introduce the basics of programming, a modified form of "dimensional flowcharting" as described by Witty[1] is utilized. This flowcharting format is accomplished by using three drawing directions for indicating sequences of actions, selection of alternative actions and refinement of larger actions into a sequence of actions. These rules are easily remembered from this picture.



Flowchart Conventions

These concepts are motivated step-by-step but we can see as the final example. the use of the rules for program-like representation of the WASH process with declarations and procedures appears in Figure 1.

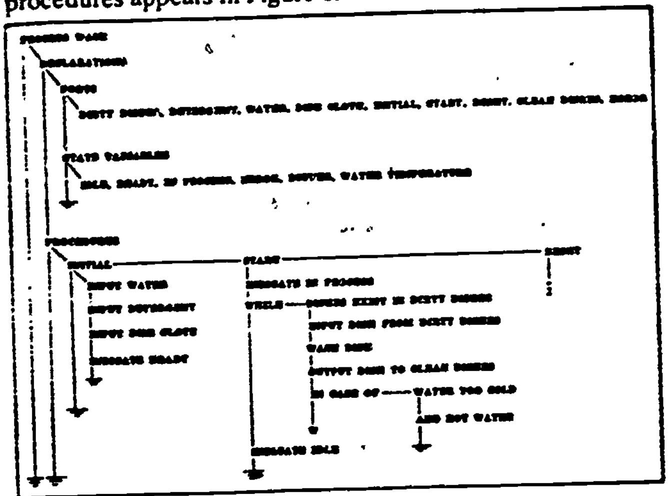


Figure 1. Flowchart of Process Logic

Your intuition should help in understanding this representation with the knowledge that the ground symbol (⏚) is used to show a sequence termination and the asterisk (*) is used to show the end of a repetitive sequence controlled by a condition at the beginning of the sequence.

At this point, enough has been presented indirectly about computing concepts in the context of the model processes that the student has achieved a basic understanding and confidence. The scope of the material described thus far has permitted the following vocabulary of concepts and terms to be understood (presented here in groupings of individual chapters).

process (task)	process inputs
process outputs	process execution
processor	system
cooperating processes	uni-processor
concurrent processes	multi-processors
interrupt	process priority
process creation	process suspension
process initiation	process termination
process resumption	processor assignment
parallel processes	

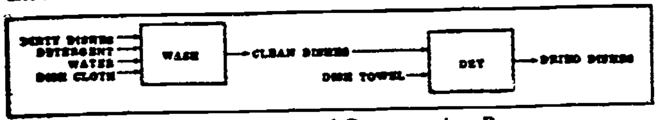
data	information
information processing system	data processing system
alphabet	storage
port	input port
output port	storage process
transmission	channels
simplex	duplex
half-duplex	

control	asynchronous control
speed (time) independent	synchronous control
speed (time) dependent	signals
error conditions	buffer
system constraints	producing process
consuming process	resource
monitor process	resource ownership
clock signal	process initialization
process (machine) state	

1) "Dimensional Flowcharting." Robert W. Witty, *Software Practice and Experience*, Vol. 7. 553-584 (1977).

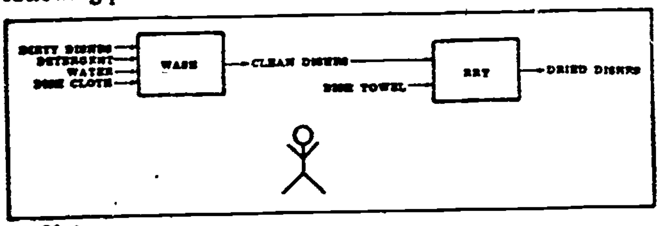
Note: Implicit in this description is the definition of a *system* as a collection of interrelated processes.

We can continue the general abstraction of this system of cooperating processes by introducing an abstraction for the second process, namely the drying process, in the following manner:



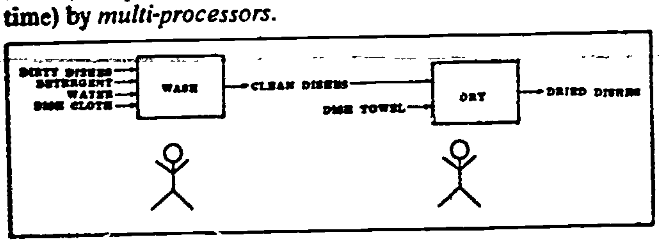
An Abstraction of a System of Cooperating Processes

If the processes are to be carried out by a single processor, *uni-processor*, then this single processor must be assigned to both the WASH and the DRY process as indicated in the following picture.



A Uni-processor

Note that if the dishrack becomes full during WASH execution, then the single uni-processor must be alternated between the execution of the processes WASH and DRY. Alternatively, we could assign a processor to each process, that is, the processes are executed *concurrently* (at the same time) by *multi-processors*.

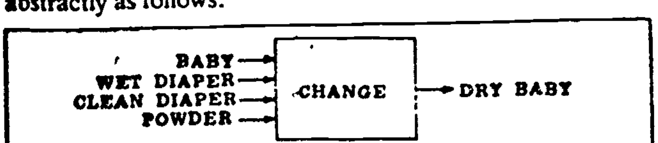


Multi-processors

During execution of a process by a processor, let us say the WASH process, the processor could be *interrupted* by a *high priority process* such as the following:

A High Priority Process

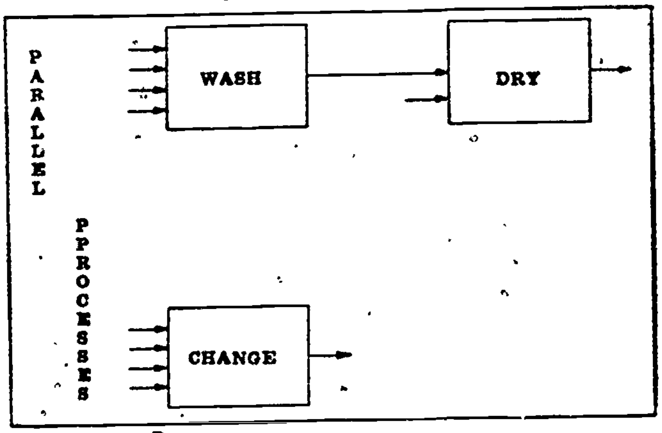
Consequently, a new process is *created* which we can represent abstractly as follows:



An Abstraction of the New Process

In the case of a single uni-processor, the processor temporarily *suspends* the process it is currently executing and *initiates* execution of the CHANGE process. After *termination* of the CHANGE process, the processor returns to, *resumes*, execution of the process that was suspended.

Alternatively, in the case of multi-processors, the processor that acknowledges the interrupt (let us say the processor serving the WASH process) could *assign* another processor (the processor assigned to DRY or another available processor) to execute the CHANGE process *parallel* with the ongoing WASH and/or DRY processes as follows:

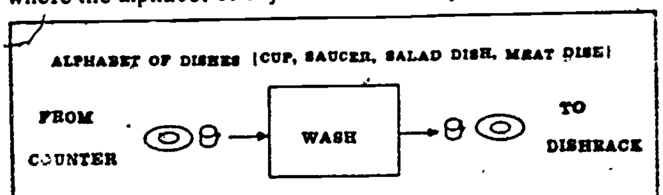


Cooperating Process

At this point, we have introduced several important computer related concepts in terms that all can understand. The flavor of this example should now be remembered as we browse through, in a highlight fashion, the further presentation of concepts and terminology. While the remaining descriptions of the approach will be terse, you can certainly use your imagination to guess how the details of the approach are presented in the book.

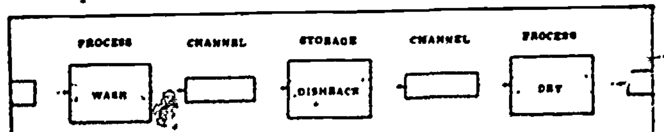
Data Flow and Control

Having introduced the basic process and system concepts, it is easy to build on the example to include data and control aspects. Data can be made analogous to *objects* processed by the processes of our example leading to the following view where the alphabet of objects is concretely *declared*.



E . eration of Process Related Objects (Data)

..e flow of objects within a process is explained in terms of ..lassical data processing steps: namely, *input* (take object from counter), *process* (wash object) and *output* (place object in dishrack). The transmission of objects between processes (over channels), including "storage processes" is indicated by the following, from which, for example, the explanations of terms such as simplex, half-duplex and full duplex are easily developed.



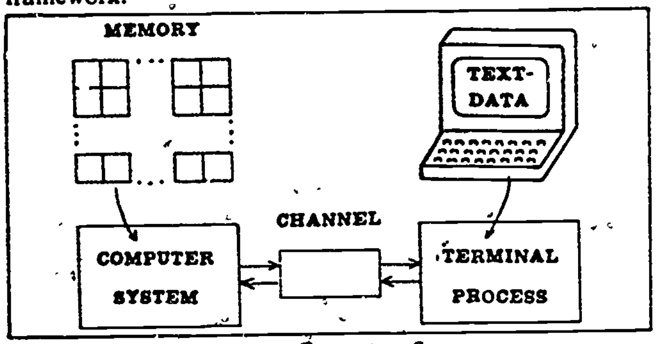
Transmission of Objects (Data) Over Channels

state transitions
finite state machine
state indicators
sequence
refinement
flowcharts (flowdiagrams)
declarations
state transition diagram
local storage
state variables
selection
program (algorithm)
repetitive sequence
procedures

How many of us mastered so many concepts in so short a period of time when we began. In classical approaches to education (including self education) in this area. many of these concepts are learned only after many hours. days. weeks, months (dare we say years) of struggling with details. It is not uncommon that the details that have been learned are then a deterrent to gaining a clear picture of these more basic, general concepts.

Introducing the Real Thing

We now begin to consider. more concretely. the data processed by computer systems by introducing the terminal and the computer system in the following. now familiar. framework.



Terminal Connection to a Computer System

It is easy to convince the student that by pushing buttons, data is transmitted across the channel and placed in a memory where it is processed. and results are sent back for display on the terminal screen. At this point. we still treat the computer as a "black box." The terminal process is used to transmit data back and forth.

In making "data" a more concrete concept. the nature of numbers of base 2 and 10 is presented including an easy-to-comprehend algorithm for the process of converting a binary number to a decimal number as indicated in Figure 2.

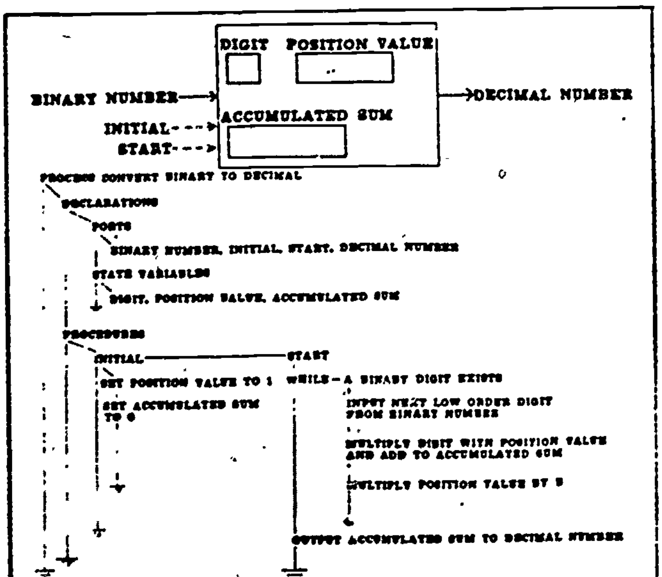


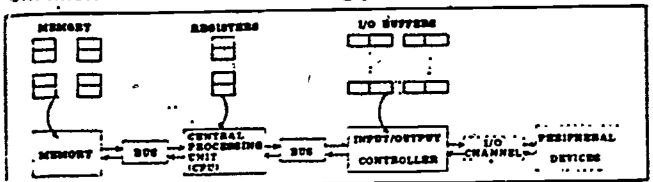
Figure 2. Binary to Decimal Conversion Process

After having mastered the WASH process from the earlier example, this process does not in any way seem strange and can be traced by following the dimensional flowchart and indicating resulting changes of the process state variables given within the process.

Further details on electronic representation of signals, precision of arithmetic data and the ASCII character set are easily introduced. Word processing systems and the representation of programs as text are followed by the presentation of basic data structure concepts including arrays.

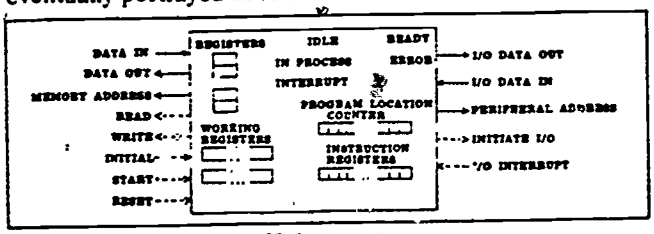
The More Conventional View

The stage has now been set to accept the computer system as simply a collection of cooperating processes connected by channels as seen in the following picture.



The Cooperating Processes of a Computer System

These constituent parts and their general properties are now easy to introduce. In fact, the CPU viewed as a process is eventually portrayed as follows:



The Central Processing Unit

A CPU state transition diagram and CPU process dimensional flowchart provide an easy guide to understanding CPU operation. A brand name independent simple instruction repertoire and assembly program presentation then makes the point about concrete programs and their execution.

Memories and Peripheral Devices

Memories of the core and semiconductor type are presented along with the structure of a flip/flop element as a process with its related process dimensional flowchart as shown in Figure 3.

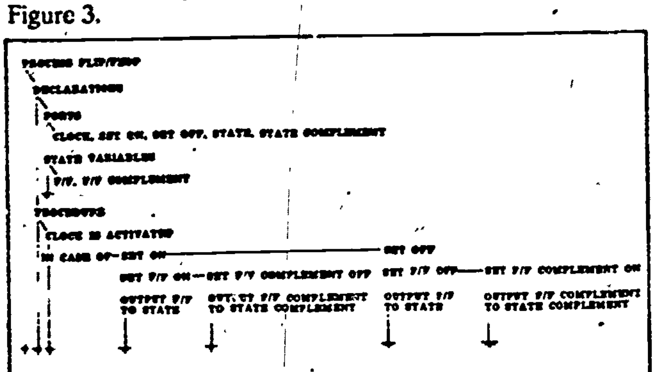
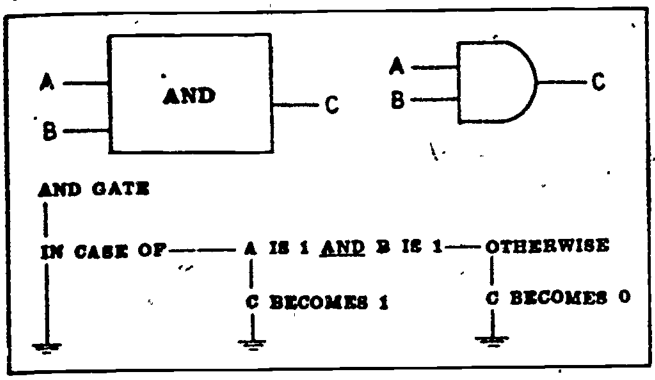


Figure 3. A Flip/Flop Process Description

Now that we are convinced that the process concept can be used as a common denominator to explain both hardware and software concepts. Storage and peripheral devices (tapes, disks, A/D and D/A converters, printers, etc.) are then described in a rather conventional manner.

Digital Processes and Hardware Construction

Even the smallest of computer elements (gates) are presented as processes with related process logic as indicated in the following example.

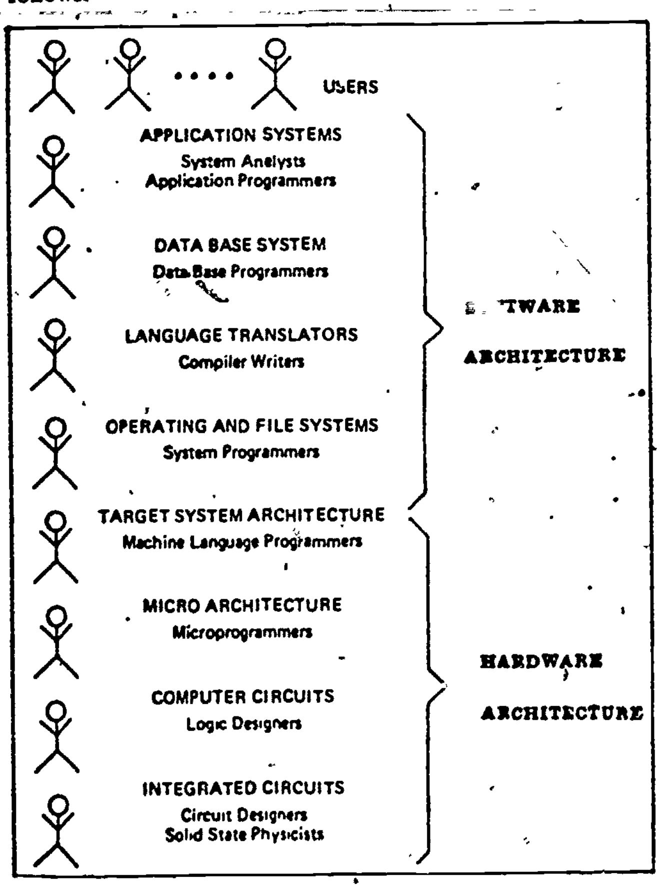


The principles of computer arithmetic are considered, including the view of a binary adder in process form. The notions of combinational and sequential circuits are conveniently introduced along with a basic explanation of clock signals and timing.

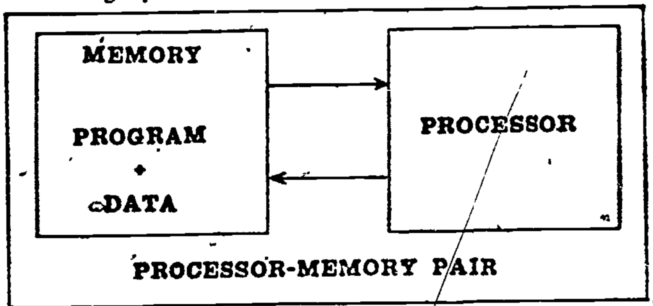
These digital building blocks and their eventual encapsulation into integrated circuits, printed circuit boards and chassis are easily described with this background.

Putting it All Together

The process concept has enabled us to present basic computer system concepts, the notion of programming and the composition of computer hardware. Putting this all together, we then create the notion of "architecture," starting with basic delineation between hardware and software architecture as follows.

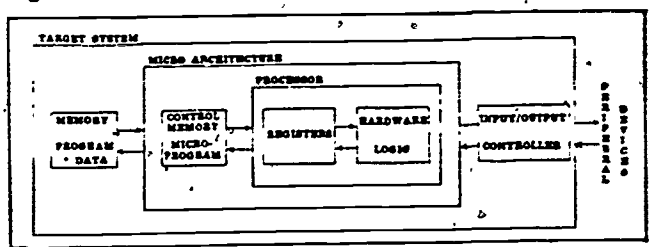


A further key concept, namely that of "processor-memory pairs" as illustrated in the following provides the keystone to describing the various architectural levels.



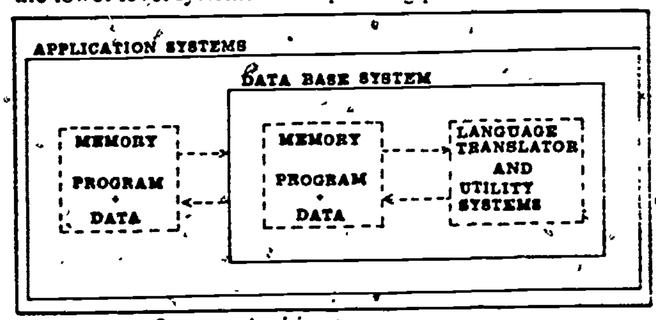
The Basis for Constructing Programmed Processes

The architecture of the target system (programmer's machine language machine) can then be described abstractly as nested pairs of processors and memories as indicated in the following.



Architecture of the Target System

This nesting is built upon to describe further the placement and role of operating and file systems, language translators, utility systems, and data bases, and culminates in the following view of the relationship between the application system and the lower level systems of cooperating processes.



Application Systems Architecture

Note: the dotted lines around the memory simply denote that it is not a separate physical memory but is a program and data representation in a common physical memory.

Further architectural concepts such as multi-port memories, DMA channels and common busses (highways) can be introduced in a natural manner as architectural alternatives. Finally, the concepts of network architectures and related terminology round off his brief (but comphehensive) introduction to the "core" concepts and terminology of computer systems.

Experience with the Approach

The approach described here has been tested and proved to be successful for a variety of audiences. For those seeking a starting point in learning about computer systems (amateurs and professionals to be) the approach permits an examination of the nature of the forest before examining the leaves of the trees. As an appreciation (computer literacy) vehicle the approach has worked successfully for journalists, politicians, and administrative and technical people. We have also been successful in removing the fear of secretaries who are learning to use word processing systems. Terminal operators of all varieties would certainly benefit from the understanding provided here. They get an idea of where they fit into the system that they are using and some idea of what is happening in the computer.

The demystification of the computer is a must if we hope to achieve any degree of harmony between technology and the citizenry of the world. This approach. which has been or is in the process of being translated into a variety of languages., provides a step in the right direction.

Some Final Comments

It is important to note that the approach is described in a completely product independent manner. therefore. the approach can be used to complement the real details of any product. Teaching assistance materials. including visual aids. are available and are quite useful for presenting the approach to small as well as large groups. The approach is well suited to being supported by animation and plans are being made to provide and animated video tape and animation via software for commonly available microcomputers.

Understanding Computer Systems can be ordered in single copies by sending a check in US dollars for $15.25 ($17.25 for airmail) to LAWSON. Gustav Adolfsgatan 9. 582. 20. Linkoping. Sweden. Make checks payable to LAWSON. All other inquires can be directed to the same address. □

CUPERTINO SCHOOL DISTRICT DEVELOPS COMPUTER LITERACY CURRICULUM

Cupertino Union School District
10301 Vista Drive
Cupertino, CA 95014
Carl W. Krause, Superintendent

Editor's Note: As more microcomputers appear in schools, more school districts or education agencies find themselves codifying goals and objectives in their computer literacy curriculum. TCT is publishing the Cupertino Union School District Computer Literacy Curriculum to promote the exchange of computer education goals and objectives. TCT solicits submission of similar curriculum plans, suggestions, critiques of this document, and reactions to TCT publishing such material.

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The Cupertino Union School District (K-8) has an average daily attendance of 13,000 pupils. Although the district is in the heart of Silicon Valley, developing and implementing a computer literacy (awareness) curriculum has taken several years. As Bobby Goodson, District Computer Resource Specialist, says:

> "The success of a program like this, introduced throughout a district, is dependent upon a well-developed inservice program with wide participation that gives teachers a good foundation to build upon. We have reached this point because we have taken time (over three years) and worked in stages. I think a district would have difficulty instituting such a program as a complete package. People need to be trained and ready with an explicit curriculum in hand if the program is to be truly successful."

District personnel and parent volunteers began working with microcomputers in 1977. Financial support came from donations, small amounts from existing budget accounts in schools, and title monies (U.S. government funds for special purposes such as programs for the gifted or handicapped). In 1981, the school board allocated capital expenditure funds to permit widespread piloting of the computer curriculum during the 1981-1982 school year. Two junior high schools will start the year with twelve microcomputers with dual floppy disk drives and one printer. Two microcomputers will be on a cart along with a 25-inch color monitor. Each elementary school that applies and is selected will receive five microcomputers, one of which will be on a cart with a 25-inch color monitor. Implementation dates will be staggered in order that staff may finish sufficient training and have adequate support as the program begins. Most schools plan to house the microcomputers in their media centers, but one plans to use them in a Specific Skills Learning Center.

Next spring the district curriculum committee will evaluate the computer literacy (awareness) curriculum and its implementation by pilot schools. The superintendent will then be in a position to make further recommendations to the board.

COMPUTER LITERACY CURRICULUM

PHILOSOPHY & GOALS OF CUSD

All students will have an opportunity to becom familiar with the operation of a microcomputer. They will become aware of the widespread use of computers in the world around them. They will be aware of both the computer's capabilities and limitations.

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IMPLEMENTATION OF THE COMPUTER CURRICULUM

In grades 3-6, the use of a microcomputer will be part of the normal school activities. It will be available for use in the classroom as part of the regular curriculum and by students for individual use. Most computer curriculum objectives at this level can be met by routine classroom use.

In grades 7-8, a specific course in Computer Awareness and Introductory Programming will give students some of the knowledge needed to make wise educational choices in high school and eventual career choices. The computer curriculum objectives can be met in such a course and/or with the inclusion of computer related topics in math, social studies and science classes.

This curriculum, a guide to the ideas students should have an opportunity to learn about computers, has been prepared with the help of many people. Without the support of the Board, administrative staff, teaching staff, and interested members of our community, this curriculum would not be possible.

The principal writer was Bobby Goodson, with the assistance of Jenny Better, Judy Chamberlin, Ron La Mar, Barbara Mumma, Jerry Prizant, Richard Pugh and Cheryl Turner.

Symbols used indicate the level at which given objectives and activities are introduced (I) and reviewed or reinforced (R).

IN SOCIAL STUDIES, STUDENTS WILL:

	K-3	4-6	7-8
1. Become familiar with a computer.			
101 — Become familiar with a microcomputer through its use in the classroom.	I	R	R
102 — Use a prepared program in a microcomputer.		I	R
103 — Describe the historical development of computing devices.		I	R
104 — Tell about the history of Silicon Valley.			I
2. Describe how computers affect our lives.			
201 — Explain ways computers affect our lives.		I	R
202 — List several ways that computers are used in everyday life.	I	R	R
203 — Identify ways that computers are used to help consumers.		I	R
204 — Illustrate the importance of the computer in modern science and industry.		I	R
205 — Identify career fields related to computer development and use.		I	R
206 — State the value of computer skills for future employment.			I
207 — Define the term "data base."			I
208 — Describe some advantages and disadvantages of a data base of personal information.			I
209 — Describe problems related to the "invasion of privacy."			I
210 — Describe ways in which computers are used to commit a wide variety of crimes and how these crimes are detected.			I
3. Describe how computers are used by social scientists.			
301 — Describe how computers are used by sociologists and other scientists.			I
302 — Describe how computer simulations are used in problem solving situations.		I	R
303 — Identify ways in which computers help make decisions.			I
304 — Explain how computer graphics are used in engineering, science, art, etc.			I

	K-3	4-6	7-8
305 — Explain how computers are used as devices for gathering and processing data.			I
306 — List several sampling techniques and statistical methods used in the social sciences.			I
307 — Describe computer applications such as those consisting of information storage and retrieval, process control, aids to decision making, computation and data processing, simulation and modeling.			I
IN LANGUAGE ARTS, STUDENTS WILL:			
4. Define and spell basic computer terms.			
401 — Define (and spell) basic computer terms.	I	R	R
5. Tell about a person or an event that influenced the historical development of computing devices.			
501 — Tell about a person or an event that influenced the historical development of computing devices.		I	R
6. Describe how computers are used in information and language related careers.			
601 — Explain the meaning of "word-processing."			I
602 — Use a computer as a word-processor.			I
603 — Describe some of the ways computers are used in the information and language related careers.			I
IN SCIENCE, STUDENTS WILL:			
7. Define "computer" and "program."			
701 — Tell what a computer is and how it works.	I	R	R
702 — Describe the historical development of computing devices as related to other scientific devices.		I	R
703 — Know the characteristics of each generation of computers.		I	R
704 — Differentiate among computers, calculators and electronic games		I	R
705 — Differentiate between analog and digital devices.			I
706 — Differentiate among micro-, mini-, and main frame computers and identify the five major components of any computer.		I	R
707 — Define (and spell) basic computer terms.	I	R	R
708 — Define software and hardware and list two examples of each.		I	R
709 — Define "computer program."		I	R
710 — Explain why a computer needs a program to operate.		I	R
711 — Define "input" and "output" and give an example of each.		I	R
712 — Recognize the relationship of a program, or input, to the result, or output.		I	R

	K-3	4-6	7-8
713 — Explain the basic operation of a computer system in terms of the input of data or information, the processing of data or information, and the output of data or information.			I
714 — Evaluate output for reasonableness in terms of the problem to be solved and the given input.		I	R
715 — Recognize the need for data to be organized to be useful.			I
716 — Describe how computers process data (searching, sorting, deleting, updating, summarizing, moving, etc.).			I
717 — State what will happen if instructions are not properly stated in the precise language for that computer.		I	R
718 — List at least three computer languages and identify the purposes for which each is used.			I
719 — State that BASIC is one of the languages used most commonly by microcomputers.		I	R
720 — Explain the existence of several variations in BASIC.		I	R
8. Explain how computers are used by scientists.			
801 — Describe the computer's place in our growing understanding of science.			I
802 — Show how a scientist would use a computer.			I
803 — Explain how computers are used in predicting, interpreting and evaluating data.			I
804 — Show that computers are best suited to tasks that require speed, accuracy and repeated operations.			I
805 — Describe situations which limit computer use (cost, availability of software and storage capacity).			I
806 — Identify common tasks which are NOT suited to computer solution.	I	R	R
807 — Explain how computer models are used in testing and evaluating hypotheses.			I
9. Use a computer to accomplish a simple task.			
901 — List several fundamental BASIC statements and commands.		I	R
902 — Differentiate between random computer commands and computer programs.		I	R
903 — State the difference between system commands and program statements.			I
904 — Use a prepared program in a microcomputer.		I	R
905 — Create a simple program in BASIC.		I	R
IN MATHEMATICS, STUDENTS WILL:			
10. Explain that the design and operation of a computer is based on standard logic patterns.			
1001 — Explain that a computer design is based on standard logic patterns.			I

	K-3	4-6	7-8
1002 — State the meaning of "algorithm."			I
1003 — Explain what is being accomplished by a given algorithm.			I
1004 — Follow and give correct output for a given algorithm.			I
1005 — Describe the standard flow chart symbols.		I	R
11. Demonstrate how a computer could be used to accomplish logical or arithmetic tasks.			
1101 — Read and explain a flow chart.		I	R
1102 — Draw a flow chart to represent a solution to a proposed problem.		I	R
1103 — Order specific steps in the solution of a problem.		I	R
1104 — Translate mathematical relations and functions into a computer program.			I
1105 — Use the computer to accomplish a mathematical task.		I	R
1106 — Evaluate output for reasonableness in terms of the problem to be solved and the given input.		I	R
1107 — State that data must be organized to be useful.			I
1108 — Describe the techniques computers use to process data such as searching, sorting, deleting, updating, summarizing, moving.			I
1109 — List several ways computers are used to process statistical data.			I
1110 — Explain the statement: "Computer mistakes" are mistakes made by people.			I
If these activities are combined into a single Junior High Computer Literacy elective, they could be regrouped with the following objectives:			
THE STUDENT WILL:			
A. Develop a vocabulary of common computer terms.			
401 (707) — Define (and spell) basic computer terms.	I	R	R
705 — Differentiate between analog and digital devices.			I
701 — Tell what a computer is and how it works.	I	R	R
704 — Differentiate among computers, calculators and electronic games.		I	R
706 — Differentiate among micro-, mini-, and main frame computers and identify the five major components of any computer.		I	R
708 — Define software and hardware and list two examples of each.		I	R
711 — Define "input" and "output" and give an example of each.		I	R
207 — Define the term "data base."			I
B. Be familiar with the history of computing devices and the development of computers.			
103 — Describe the historical development of computing devices.		I	R
702 — Describe the historical development of computing devices as related to other scientific devices.		I	R

	K-3	4-6	7-8
501 — Tell about a person or an event that influenced the historical development of computing devices.		I	R
104 — Tell about the history of Silicon Valley.			I
703 — List the characteristics of each generation of computers.		I	R
C. Develop an understanding of how computers are used.			
201 — Explain ways computers affect our lives.		I	R
202 — List several ways that computers are used in everyday life.	I	R	R
203 — Identify ways that computers are ~~used to~~ help consumers.		I	R
302 — Describe how computer simulations are used in problem solving situations.		I	R
303 — Identify ways in which computers help make decisions.			I
803 — Explain how computers are used in predicting, interpreting and evaluating data.			I
807 — Explain how computer models are used in testing and evaluating hypotheses.			I
1109 — List several ways computers are used to process statistical data.			I
306 — List several sampling techniques and statistical methods used in the social sciences.			I
601 — Explain the meaning of "word processing."			I
801 — Describe the computer's place in our growing understanding of science.			I
204 — Illustrate the importance of the computer in modern science and industry.		I	R
304 — Explain how computer graphics are used in engineering, science, art, etc.			I
305 — Explain how computers are used as devices for gathering and processing data.			I
307 — Describe computer applications such as those consisting of information storage and retrieval, process control, aids to decision making, computation and data processing, simulation and modeling.			I
804 — Show that computers are best suited to tasks that require speed, accuracy and repeated operations.			I
805 — Describe situations which limit computer use (cost, availability of software and storage capacity).			I
208 — Describe some advantages and disadvantages of a data base of personal information.			I
209 — Describe problems related to the "invasion of privacy."			I
210 — Describe ways in which computers are used to commit a wide variety of crimes and how these crimes are detected.			I
D. Learn about computer-related career opportunities.			
205 — Identify career fields related to computer development and use.		I	R

	K-3	4-6	7-8
206 — State the value of computer skills for future employment.			I
301 — Describe how computers are used by sociologists and other social scientists.			I
603 — Describe some of the ways computers are used in the information and language related careers.			I
802 — Show how a scientist would use a computer.			I
E. Gain a non-technical understanding of how computers function.			
1001 — Explain that a computer design is based on standard logic patterns.			I
713 — Explain the basic operation of a computer system in terms of the input of data or information, the processing of data or information, and the output of data or information.			I
716 — Describe how computers process data (searching, sorting, deleting, updating, summarizing, moving, etc.).			I
1108 — Describe the techniques computers use to process data such as searching, sorting, deleting, updating, summarizing, moving.			I
715 (1107) — Recognize the need for data to be organized to be useful.			I
1110 — Explain the statement: "Computer mistakes" are mistakes made by people.			I
806 — Identify common tasks which are NOT suited to computer solution.	I	R	R
709 — Define "computer program."		I	R
710 — Explain why a computer needs a program to operate.		I	R
712 — Recognize the relationship of a program, or input, to the result, or output.		I	R
714 (1106) — Evaluate output as to its reasonableness in terms of the problem to be solved and the given input.		I	R
717 — State what will happen if instructions are not properly stated in the precise language for that computer.		I	R
718 - List at least three computer languages and identify the purposes for which each is used.			I
719 — State that BASIC is one of the languages used most commonly by microcomputers.		I	R
720 — Explain the existence of several variations in BASIC.		I	R
F. Learn to use a computer.			
101 — Become familiar with a microcomputer through its use in the classroom.	I	R	R
102 (904) — Use a prepared program in a microcomputer.		I	R
1002 — State the meaning of "algorithm."			I
1003 — Explain what is being accomplished by a given algorithm.			I
1004 — Follow and give correct output for a given algorithm.			I
1005 — Describe the standard flow chart symbols.		I	R
1101 — Read and explain a flow chart.		I	R

	K-3	4-6	7-8
1102 — Draw a flow chart to represent a solution to a proposed problem.		I	R
1103 — Order specific steps in the solution of a problem.		I	R
901 — List several fundamental BASIC statements and commands.		I	R
902 — Differentiate between random computer commands and computer programs.		I	R
903 — State the difference between system commands and program statements.			I
905 — Create a simple program in BASIC.		I	R
1105 — Use the computer to accomplish a mathematical task.		I	R
1104 — Translate mathematical relations and functions into a computer program.			I
602 — Use a computer as a word-processor.			I

END

Section IX

COMPUTER SIMULATIONS

in

MATHEMATICS

Computer Simulations in Mathematics Education

By Janice L. Flake

Janice L. Flake, Associate Professor of Mathematics Education, Florida State University, Tallahassee

Computer simulations can be used to make theory more relevant. Such simulations can be used for making either: (1) mathematical theory relevant for the youngster, or (2) educational theory relevant for the teacher education student.

Background

A simulation is a working analogy of a situation, such as a "flight simulator" or "classroom simulator." Through gaming activities, the student can manipulate the environment to study the interactions of various conditions. The idea is to put the student in the middle of a simulated problem situation; through manipulating the situation, he is to solve the problem.

Such activities should provide for the following stages of learning: (1) exploratory investigations, (2) formulating of principles, and (3) operating from principles. Thus, the same simulation could be used to: (1) introduce the theory, (2) provide for avenues for discovery of principles, and (3) provide for reinforcement of the principles.

There are four basic components in a simulation game: (1) an abstracton of an environment—this is the model, perhaps consisting of a system of models, (2) a series of rules for how the model behaves, or models interact—this is the simulation, (3) the freedom for the student to interact with the simulation to develop his or her own strategies—this is the game, and (4) "reality" feedback—this is what makes it come "alive."

A simulation study begins with the development of a custom-made model. Such a model may employ a systems approach, where a system is a group of interdependent elements acting together to accomplish a predetermined purpose. For example, several educational theories, e.g., teaching strategies and questioning behaviors, can be written into the model.

Rationale

Too often theory is given primarily in the abstract form, and students fail to see its relevance. Simulations can put the student into a concrete example of the theory. As the student manipulates the simulated environment, he should gain insights as to how the theory relates.

Having the facilities of an interactive computer system allows for immediate feedback. Hence, the student can explore a number of possibilities within a matter of a few minutes. Through his exploratory investigations, he can find some patterns for behavior of the model. Those patterns can help him to become aware of relationships.

In addition, the students tend to regard what they are doing as play. There are a number of strong supporters of the merits of play in schools (Piaget, 1951; Bruner, 1975; Caplan and Caplan, 1974).

Simulations can also be used to study individual behaviors. The way the student manipulates the environment can give a great deal of information concerning his or her thinking patterns. This information can unobtrusively be stored and used to study the child's thinking patterns, and thus can lead to research.

Examples

The following examples show how such simulations could be used throughout the curriculum. Most of the examples relate to a computer system such as PLATO: This

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system offers the facilities of graphics and animated motion. The visual effects add much to the dynamics of the situation, i.e., they bring much "action" to the student. Additional features that can be used for little children are audio facilities, and the touch panel, where the child simply touches the screen and the location is recorded.

Example 1

At present I am working with some 6 year olds, trying to teach them the concept of linear measure. The literature suggests that youngsters of this age have difficulty focusing on lengths. Thus, we are using Cuisenaire rods to "build roads." The students are trying to find who can make the longest road. This activity has kept the children's attention for several days.

A simulation of this activity can be developed for PLATO, where, again, the children are trying to make the longest road, or perhaps help a playmate cross a ravine to get to a friend. (See Figure 1.)

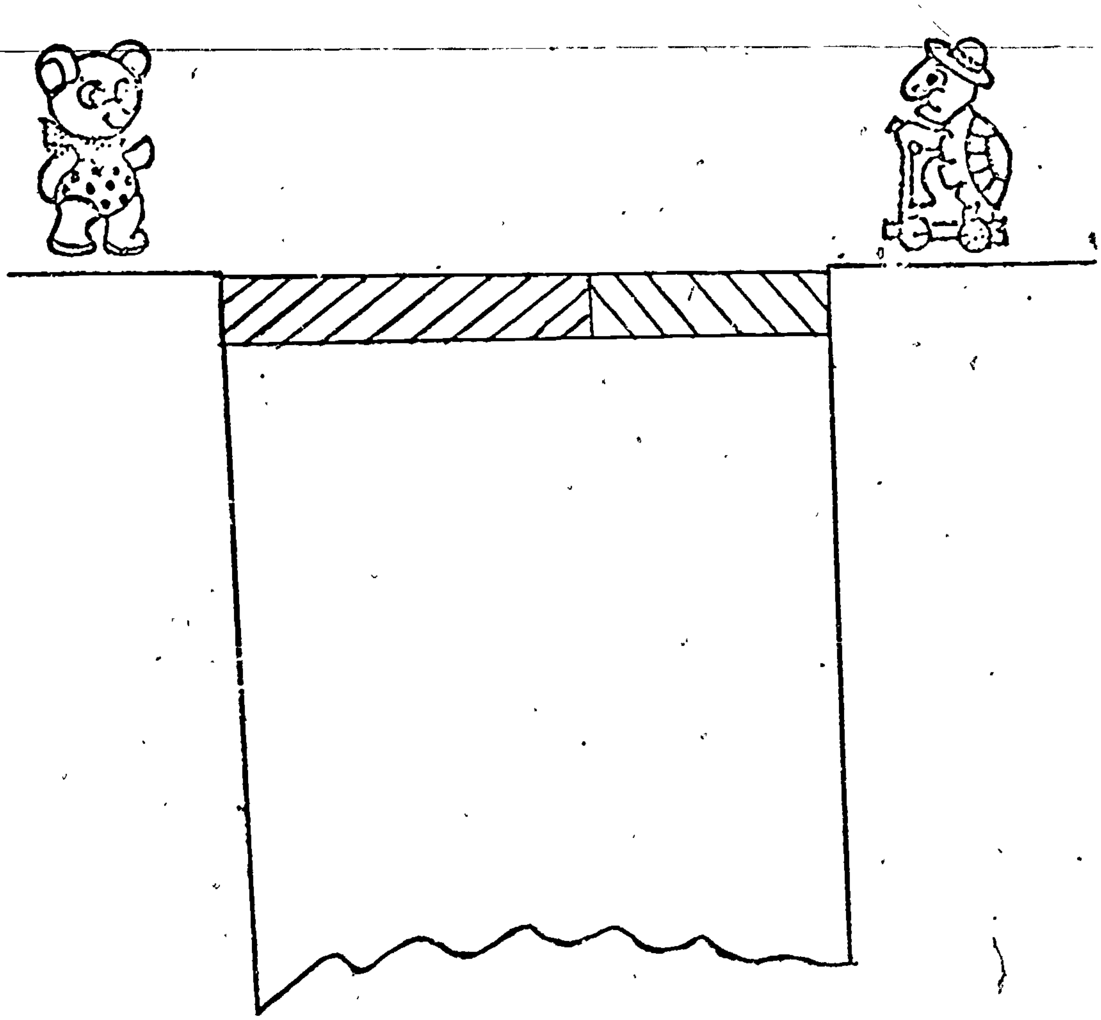

Figure 1.

Within certain constraints, e.g., time limits, the student is to decide which replicas of rods he wishes to use. Using the touch panel, the child can "move" the rods to form his road. If the youngster is successful in getting the road built, then the playmate runs across the road to his friend. If the youngster uses an ineffective strategy and does not build an appropriate road, then the road tumbles into the ravine.

Through manipulation of this game, the youngster will find an algorithm of putting the longest rods first, then the next longest rods, and so on. Hence, giving the youngster the freedom to manipulate the game can allow him to go through the various stages of learning and build his own strategies.

The use of the simulated activity is more dynamic than the straight use of the blocks because. (1) more controls can be set and a greater amount of constraints can be placed, (2) the youngster can receive dynamic feedback, which, hopefully, will keep his or her attention, and (3) accurate records of the child's attempts can be kept. Through the study of such records one can tell if the child is improving his strategy.

Such a program could be used with first or second graders. The use of the actual rods, though, should be a prerequisite to the use of this program. Piaget (1956) warns that it is the actual "action" of manipulating the blocks that helps build concepts.

Example 2

The number line is rich with possibilities for simulations. Through taking trips, basic operations of addition, subtraction, multiplication, and division can be evolved. One can first build ideas of these basic operations with whole numbers, then carry the same idea through to the treatment of rational numbers.

PLATO has the additional feature that a trail can be left showing the path that has been traveled. Thus, if one were traveling from 3 to 5, the student could observe the motion of an object traveling from 3 to 5 with a trail left, showing the traveled path. The same program can also be written at a more abstract level where no trail is left.

The use of the number line can allow for a much more consistent introduction to fractional numbers than is normally done. Once the child has the unit iteration idea, he can treat a number such as $\frac{3}{4}$ as 3 one-fourth units. This would avoid the often confusing use of many different interpretations of fractional numbers, such as parts out of parts, parts of a whole, and so on.

Caution should be used, however, in using the number line prematurely. The number line is based upon perception. PLATO is a highly perceptual instrument. Piaget (1956) warns that a child's conception of space does not evolve out of perceptions alone. The child needs considerable actions on objects (manipulative aids) to build concepts. The extremely important concept of linear measure should be a prerequisite for using the number line to illustrate operations on numbers. I would not want to use the number line with children until they have internalized the coordination of the reference system.

Example 3

"Rate-distance-time" problems often give youngsters difficulty. Consider a simulation of two cars driving on two intersecting highways. Allow the child to manipulate various values for rate, distance, and time to see if he or she can make the cars avoid each other or crash into each other. The action then is actually carried out. To accentuate the effects of one of the variables, one might give fixed values to two of them, and vary the values of the other.

This program could be used at the fourth grade level or higher.

Example 4

The ideas of quadratic functions can be illustrated through the use of motion properties. For example, suppose one shoots a basket with a basketball. One can vary the initial velocity and initial angle until he finds the right combination.

Such a problem could be used at the exploratory investigation stage of learning to introduce a unit on quadratic functions. Then it could be used to help build properties of quadratic functions. Later, it could be used

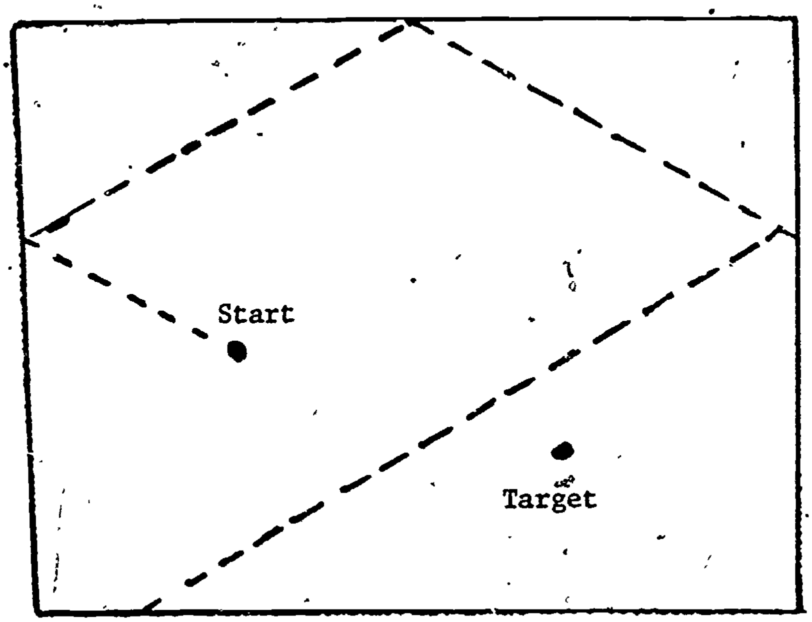

Figure 2.

to reinforce the properties, where the student is operating at the application of principles level of learning.

Example 5

The ideas of line reflections can be illustrated through a simulated billiards game. The student is to "shoot" a target from a starting location, and reflect his simulated ball off of each of the walls. (See Figure 2.)

The student sees the action carried out, and can try over again and again until he has developed a strategy that works. The starting position and the target can be randomly placed, allowing more flexibility.

Example 6

Calculus is rich with many applications where the use of graphics is very effective. Problems of maximum/minimum motion, instantaneous velocity, and area/volume are a few of the possibilities that could be simulated very easily.

Example 7

An example of educational theory being made relevant for teacher education interns can be in the form of interns "teaching" simulated classes. The intern is to "ask" the class a question, get a simulated response, react to that response, "ask" a next question, and so on. Through such questioning behaviors, the intern is to "teach" the class a principle or concept.

Such a simulation is described in more detail in other publications (Flake, 1973, 1974). In this simulation articulation of several educational models was employed: (1) lesson planning, (2) Henderson's moves and strategies of teaching mathematics (Henderson, 1963, 1967), (3) Wills' approach to problem solving (Wills, 1967), (4) a simplified learning theory, and (5) various questioning behaviors.

Additional Examples

The mathematical examples given above are rather simple; far more complex problems can be created. For example, the study of ecology, economics, traffic problems using linear programming techniques, agricultural planning, and flight patterns all lend themselves to interesting simulations. The educational simulation briefly mentioned in example 7 is an example of a far more complex simulation, where some components of a classroom interaction are studied.

Capable secondary level students can become quite proficient in programming, and delight in creating their own simulations. This is an excellent experience for them, because in order to create a simulated environment, one must carefully understand the environment that is to be simulated.

Summary

Some principles have been given concerning how one could use simulations as a part of the mathematics curriculum. Some examples have been given. These are only a few of the many possible examples that could be given. It is hoped that simulations can give relevance, motivation, and a deeper understanding of the theory, as well as lead to useful research.

Let's get some "action" into mathematics, and help mathematics become more relevant!

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PROBABILITY SIMULATION IN MIDDLE SCHOOL

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By middle-school age many students have formed notions about the likelihood of an event happening. These notions are often incorrect, and yet firmly fixed in the minds of students. Studying probability as a set of theoretical rules, in our experience, is very unlikely to change any of these preconceived notions. Teaching probability through concrete experiments holds much more promise.

We will describe two "situations" that are real to students—basketball and cereal box prizes. Each situation is simulated by a spinner against an appropriate background. The "realness" of the physical situation is not lost; exclamations of "Boy, you missed again," and "Now you've only got to get the princess," show the involvement of the students as they experiment and gather data.

Basketball Simulation

The situation simulated here (proposed by the Mathematics-Methods Project at Indiana University) is the one-and-one foul shot situation in basketball. A player making a free throw from the foul line is given a second shot only if on the first, the ball goes through the hoop. Thus, a basketball player shooting a one-and-one can score 0, 1, or 2 points.

Suppose we know a player shoots with 60 percent accuracy. We ask students, "In 25 trips to the foul line for a one-and-one, how many times will the player get 2 points, 1 point, and 0 points?" Most students guess that 1 point will happen more often than either 0 points or 2 points.

The attempt to make a foul shot can be simulated by a spin against the background shown in figure 1. If the first "throw" results in a basket, another spin is made. One group of students obtained the data in table 1. At first, there appears to be little pattern in the results. However, after doing four sets of twenty-five shots, most students acknowledge that 1 point is not occurring as frequently as 0 or 2.

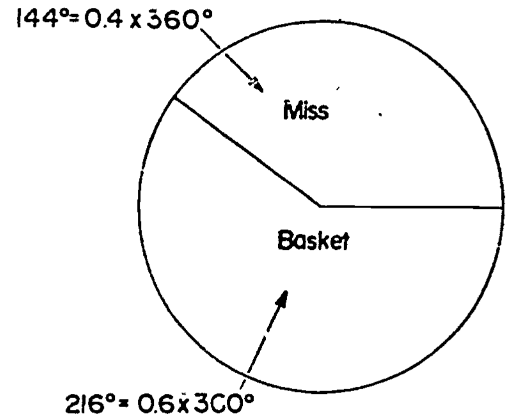

Fig. 1. Background for player who shoots with 60 percent accuracy

From a mathematical point of view, this is a very interesting situation because the theoretical probabilities of the events 0 points, 1 point, and 2 points are obtained by multiplying probabilities, which is always harder for students to understand than situations where probabilities are added. The theoretical expected frequencies, computed from the probabilities of a basket, 0.6, or a miss, 0.4, are given in table 2.

Combining the class results and writing the frequency fractions as decimals give results close to these theoretical values. Because the numbers 0.4 and 0.36 are close, it is often difficult to detect a difference between the frequencies for 0 points and for 2 points.

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TABLE 1
One Group's Results of Four Trials

Points	Trial 1	Trial 2	Trial 3	Trial 4	Total	Frequency
0	7	10	11	4	32	$\frac{32}{100} = 0.32$
1	4	3	4	10	21	$\frac{21}{100} = 0.21$
2	14	12	10	11	47	$\frac{47}{100} = 0.47$

We can observe a different player who has probability 0.7 of making a free throw by using a new background for the spinner (see fig. 2). The theoretical expected fre-

TABLE 2
Theoretical Frequencies for 60 Percent

Points	Probability	
0	0.4	= 0.4
1	0.6 × 0.4	= 0.24
2	0.6 × 0.6	= 0.36

quencies for 0, 1, and 2 points are given in table 3. The students' results will now almost certainly show a clear distinction between the frequencies for 0 points and for 2 points.

TABLE 3
Theoretical Frequencies for 70 Percent

Points	Probability	
0	0.3	= 0.3
1	0.7 × 0.3	= 0.21
2	0.7 × 0.7	= 0.49

After simulating the results for two players, many students will make reasonable predictions of the results for a superstar who shoots with 90 percent accuracy. A follow-up with older or more able students would be an investigation of the theoretical frequencies—why probabilities are multiplied, and so on.

In the previous activity, each group's experimental results agreed, on the whole, with the averages of the class results. In the next activity, the class results are usually quite different from the results of any student. This emphasizes the point that probabilistic statements are statements about what will happen in the long run, over many trials of the experiment.

Star Wars Simulation

The goal for the students is to accumulate a complete set of six *Star Wars* models by "buying" cereal boxes. One model is contained in each box. (We are assuming there are so many boxes available that each box is equally likely to contain any one of the models.) The students' guesses of how many boxes they would, on the average, have to buy to get a complete set are usually too small. We can simulate purchasing a box by a spin using a background for the spinner that reflects the situation—six equally likely outcomes (see fig. 3). The students keep count of how many boxes are bought in collecting a complete set of models. We had each group record their data

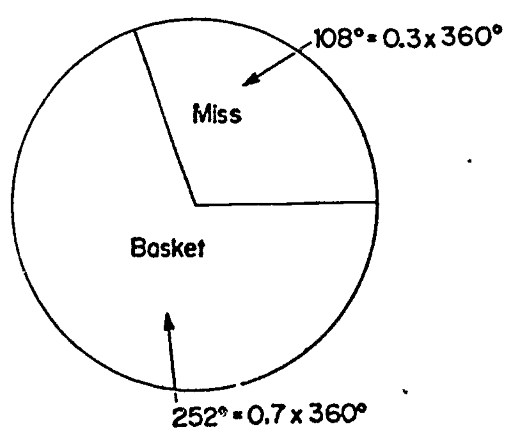

Fig. 2

for collecting four complete sets of models (see table 4).

The theoretical or expected number of boxes is obtained using the fact that if an event has probability p, on the average it will take $1/p$ trials for it to occur. For example, the average number of tosses of a coin until heads occurs is $1/0.5 = 2$. For the models, the probability that the second box contains a different figure from the first is 5/6. Once two distinct models have been collected, the probability of a box having a model different from both is 4/6. The expected number of boxes is thus the sum of

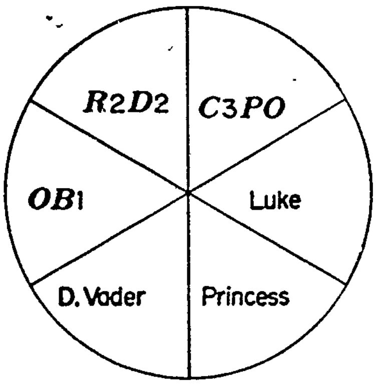

Fig. 3

$$1 + \frac{1}{\frac{5}{6}} + \frac{1}{\frac{4}{6}} + \frac{1}{\frac{3}{6}} + \frac{1}{\frac{2}{6}} + \frac{1}{\frac{1}{6}}$$
$$= 6\left(\frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1\right) = 14.7.$$

The students listed the numbers in the last column (total number of spins) on the board. They varied widely: on a list of twenty-five numbers, there may be one greater than 45 and another as small as 8. The average will be close to 15. A discussion of the significance of this average will provide a background for the ideas of dispersion and deviation. How representative is the average? Are half the numbers listed within 2 or 3 of this average? Are 2/3 of them within 5 or 6 of the average?

Related investigations would be for the students to construct new backgrounds for the spinners and compute the average number of boxes needed if there are two, three, four, five, six, seven, or eight different models in the set. The expected averages for these numbers are given in table 5. For seven different figures the average number of boxes is

$$7\left(\frac{1}{7} + \frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1\right) = 18.15.$$

If the students graph the number of models versus the average number of boxes, they may be able to predict the average for twenty-five different models (baseball cards) by extrapolating from the graph. After this much hands-on simulation, it would be appropriate to replace the spinner with a computer program to do the simulation and to keep track of the results. A sample output from the BASIC program at the end of this article is given in table 6

TABLE 4

Set #1	Darth Vadar	OB1 Kenobi	R2D2	C3PO	Luke	Princess	Total Number of Spins
1	II	卌 I	卌 卌	卌	I	卌	29
2	II	IIII	卌 I	I	IIII	III	20
3	卌	IIII	卌 IIII	I	I	IIII	24
4	III	I	II	I	I	卌	13

TABLE 5
Approximate Expected Averages

Number of models	2	3	4	5	6	7	8
Approximate expected average	3	5.5	8.3	11.41	14.7	18.15	21.74

(both have been modified for publication purposes).

TABLE 6
Sample Computer Output

```
RUN
 BASEBALL CARDS.  THERE ARE
N CARDS IN THE SERIES. ENTER N
25
FROM HOW MANY TRIALS DO YOU
WANT TO COMPUTE THE AVERAGE?
10
 69 CARDS
 151 CARDS
 137 CARDS
 108 CARDS
 108 CARDS
 88 CARDS
 133 CARDS
 141 CARDS
 48 CARDS
 101 CARDS
AVERAGE NUMBER OF CARDS IS 108.4
RUN
 BASEBALL CARDS.  THERE ARE
N CARDS IN THE SERIES. ENTER N
25
FROM HOW MANY TRIALS DO YOU
WANT TO COMPUTE THE AVERAGE?
10
 82 CARDS
 198 CARDS
 74 CARDS
 192 CARDS
 67 CARDS
 86 CARDS
 140 CARDS
 56 CARDS
 90 CARDS
 127 CARDS
AVERAGE NUMBER OF CARDS IS 111.2
```

Conclusion

Both of these activities were designed to involve students in exploring a probabilistic situation about which their intuition was faulty. Each of these is rich from the point of view both of mathematical thinking and processes and of the potential for spin-off in interesting and challenging directions. Making conjectures, modeling a situation, gathering data, and validating the conjectures should be a part of every student's mathematical experiences.

APPENDIX

BASIC Program for Baseball Cards

```
100 DIM C(100)
110 PRINT " BASEBALL CARDS. THERE ARE "
115 PRINT "N CARDS IN THE SERIES. ENTER N"
120 INPUT N
130 PRINT "FROM HOW MANY TRIALS DO YOU "
135 PRINT "WANT TO COMPUTE THE AVERAGE?"
140 INPUT K
150 S = 0
160 FOR I = 1 TO K
170 D = 0
180 FOR J = 1 TO N
190 C(J) = 0
200 NEXT J
210 X = INT (N*RND(-1)) + 1
220 C(X) = C(X) + 1
230 IF C(X) = 1 THEN 250
240 GO TO 210
250 D = D + 1
260 IF D = N THEN 280
270 GO TO 210
280 T = 0
290 FOR J = 1 TO N
300 T = T + C(J)
310 NEXT J
320 PRINT T; " CARDS"
330 S = S + T
340 NEXT I
350 PRINT "AVERAGE NUMBER OF CARDS IS";S/K
360 END
```